Ohio | Department of Education

Ohio's State Tests

ITEM RELEASE

SPRING 2018

GEOMETRY

Table of Contents

Geometry
Spring 2018 Item Release
Content Summary and Answer Key

Question No.*	Item Type	Content Cluster	Content Standard	Answer Key	Points
4	Multiple Choice	Prove theorems involving similarity.	Prove theorems about triangles. *Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity. (G.SRT.4)*	A	1 point
8	Equation Item	Define trigonometric ratios and solve problems involving right triangles.	Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles. *(G.SRT.6)*	---	1 point
9	Equation Item	Use coordinates to prove simple geometric theorems algebraically.	Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point). *(G.GPE.5)*	---	1 point
11	Equation Item	Apply geometric concepts in modeling situations.	Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot). *(G.MG.2)*	---	1 point

** The question number matches the item number in the Item Level Report in the Online Reporting System. The items are numbered sequentially in the practice site.*

Geometry
Spring 2018 Item Release
Content Summary and Answer Key

Question No.*	Item Type	Content Cluster	Content Standard	Answer Key	Points
12	Equation Item	Use the rules of probability to compute probabilities of compound events in a uniform probability model.	Find the conditional probability of *A* given *B* as the fraction of *B*'s outcomes that also belong to *A*, and interpret the answer in terms of the model. *(S.CP.6)*	---	1 point
15	Equation Item	Use the rules of probability to compute probabilities of compound events in a uniform probability model.	Apply the Addition Rule, $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$, and interpret the answer in terms of the model. *(S.CP.7)*	---	1 point
16	Multiple Choice	Visualize relationships between two-dimensional and three-dimensional objects.	Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects. *(G.GMD.4)*	A	1 point
18	Equation Item	Understand independence and conditional probability and use them to interpret data.	Understand that two events *A* and *B* are independent if the probability of *A* and *B* occurring together is the product of their probabilities, and use this characterization to determine if they are independent. *(S.CP.2)*	---	1 point

** The question number matches the item number in the Item Level Report in the Online Reporting System. The items are numbered sequentially in the practice site.*

Geometry
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Content Summary and Answer Key

Question No.*	Item Type	Content Cluster	Content Standard	Answer Key	Points
19	Equation Item	Understand and apply theorems about circles.	Identify and describe relationships among inscribed angles, radii, and chords. *Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle. (G.C.2)*	---	1 point
21	Hot Text	Prove geometric theorems.	Prove theorems about parallelograms. *Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals. (G.CO.11)*	---	1 point
23	Equation Item	Define trigonometric ratios and solve problems involving right triangles.	Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems. *(G.SRT.8)*	---	1 point
29	Equation Item	Find arc lengths and areas of sectors of circles.	Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector. *(G.C.5)*	---	1 point

** The question number matches the item number in the Item Level Report in the Online Reporting System. The items are numbered sequentially in the practice site.*

Geometry
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Content Summary and Answer Key

Question No.*	Item Type	Content Cluster	Content Standard	Answer Key	Points
31	Multiple Choice	Experiment with transformations in the plane.	Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc. *(G.CO.1)*	D	1 point
34	Equation Item	Explain volume formulas and use them to solve problems.	Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems. *(G.GMD.3)*	---	1 point
39	Hot Text	Prove geometric theorems.	Prove theorems about lines and angles. *Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints. (G.CO.9)*	---	1 point
40	Equation Item	Define trigonometric ratios and solve problems involving right triangles.	Explain and use the relationship between the sine and cosine of complementary angles. *(G.SRT.7)*	---	1 point
44	Hot Text	Prove geometric theorems.	Prove theorems about triangles. *Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point. (G.CO.10)*	---	2 points

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Geometry
Spring 2018 Item Release
Content Summary and Answer Key

Question No.*	Item Type	Content Cluster	Content Standard	Answer Key	Points
45	Equation Item	Understand similarity in terms of similarity transformations.	Verify experimentally the properties of dilations given by a center and a scale factor: *(G.SRT.1)* *a.* A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.	---	2 points
47	Multiple-Select	Understand congruence in terms of rigid motions.	Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent. *(G.CO.6)*	C, D, E	1 point
48	Table Input	Understand independence and conditional probability and use them to interpret data.	Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. *For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results. (S.CP.4)*	---	1 Point

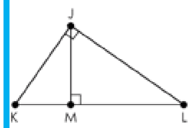
** The question number matches the item number in the Item Level Report in the Online Reporting System. The items are numbered sequentially in the practice site.*

Geometry
Spring 2018 Item Release

Question 4

Question and Scoring Guidelines

Question 4



Mark is proving the Pythagorean Theorem. He draws right triangle JKL with altitude $\overline{JM}$. First he proves $\triangle JKL \sim \triangle MKJ$ and $\triangle JKL \sim \triangle MJL$ using the Angle-Angle criterion. The rest of his proof is shown with some steps missing.

Statements	Reasons
1. $\triangle JKL \sim \triangle MKJ$ and $\triangle JKL \sim \triangle MJL$	1. Angle-Angle criterion
2. $\frac{JK}{LK} = \frac{MK}{JK}$ and $\frac{LJ}{LK} = \frac{ML}{LJ}$	2. Corresponding sides of similar triangles are proportional
3. $(JK)^2 = LK \bullet MK$ and $(LJ)^2 = LK \bullet ML$	3. Multiplication property of equality
4.	4.
5.	5.
6. $MK + ML = LK$	6. Segment addition postulate
7. $(JK)^2 + (LJ)^2 = (LK)^2$	7. Substitution

Which two steps are missing from the proof?

Ⓐ

4. $(JK)^2 + (LJ)^2 = LK \cdot MK + LK \cdot ML$	4. Addition property of equality
5. $(JK)^2 + (LJ)^2 = LK(MK + ML)$	5. Distributive property

Ⓑ

4. $(JK)^2 + (LJ)^2 = LK \cdot MK + LK \cdot ML$	4. Addition property of equality
5. $(JK)^2 + (LJ)^2 = LK(LK + LK)$	5. Distributive property

Ⓒ

4. $(JK)^2 \cdot (JK)^2 = LK \cdot MK \cdot LK \cdot ML$	4. Multiplication property of equality
5. $(JK)^2 \cdot (LJ)^2 = LK(MK \cdot ML)$	5. Distributive property

Ⓓ

4. $(JK)^2 \cdot (JK)^2 = LK \cdot MK \cdot LK \cdot ML$	4. Multiplication property of equality
5. $(JK)^2 \cdot (LJ)^2 = LK(LK \cdot LK)$	5. Distributive property

Points Possible: 1

Content Cluster: Prove theorems involving similarity.

Content Standard: Prove theorems about triangles. *Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity. (G.SRT.4)*

Scoring Guidelines

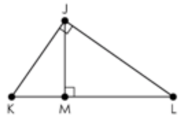
Rationale for Option A: **Key** – The student correctly realizes that adding the squares of the lengths of the two legs of triangle JKL is necessary to complete the proof. Applying the Addition Property of Equality to combine the left sides and right sides of the two equations creates the correct equation in step 4. Factoring LK out on the right side (using the Distributive Property) leads to the expression LK(MK + ML), which results in LK • LK after using the Segment Addition Postulate, and concludes the proof of Pythagorean Theorem.

Rationale for Option B: The student correctly identifies that the Addition Property of Equality should be used to combine the left sides and the right sides of the two equations to create the correct equation for step 4 but may miss that factoring LK out in step 5 would result in LK • (LK + LK) = LK^2, which is not equivalent to the expression in step 6.

Rationale for Option C: The student may think that step 4 involves multiplication instead of addition and then incorrectly factor the expression on the right side to obtain the expression in step 6.

Rationale for Option D: The student may think that step 4 involves multiplication instead of addition and only the length of LK would be on the right sides of step 5, or LK(LK • LK), that is equal to LK^3 but not LK^2, as stated in Step 7.

Sample Response: 1 point



Mark is proving the Pythagorean Theorem. He draws right triangle JKL with altitude $\overline{JM}$. First he proves $\triangle JKL \sim \triangle MKJ$ and $\triangle JKL \sim \triangle MJL$ using the Angle-Angle criterion. The rest of his proof is shown with some steps missing.

Statements	Reasons
1. $\triangle JKL \sim \triangle MKJ$ and $\triangle JKL \sim \triangle MJL$	1. Angle-Angle criterion
2. $\frac{JK}{LK} = \frac{MK}{JK}$ and $\frac{LJ}{LK} = \frac{ML}{LJ}$	2. Corresponding sides of similar triangles are proportional
3. $(JK)^2 = LK \cdot MK$ and $(LJ)^2 = LK \cdot ML$	3. Multiplication property of equality
4.	4.
5.	5.
6. $MK + ML = LK$	6. Segment addition postulate
7. $(JK)^2 + (LJ)^2 = (LK)^2$	7. Substitution

Which two steps are missing from the proof?

● (A)

4. $(JK)^2 + (LJ)^2 = LK \cdot MK + LK \cdot ML$	4. Addition property of equality
5. $(JK)^2 + (LJ)^2 = LK(MK + ML)$	5. Distributive property

Ⓑ

4. $(JK)^2 + (LJ)^2 = LK \cdot MK + LK \cdot ML$	4. Addition property of equality
5. $(JK)^2 + (LJ)^2 = LK(LK + LK)$	5. Distributive property

Ⓒ

4. $(JK)^2 \cdot (JK)^2 = LK \cdot MK \cdot LK \cdot ML$	4. Multiplication property of equality
5. $(JK)^2 \cdot (LJ)^2 = LK(MK \cdot ML)$	5. Distributive property

Ⓓ

4. $(JK)^2 \cdot (JK)^2 = LK \cdot MK \cdot LK \cdot ML$	4. Multiplication property of equality
5. $(JK)^2 \cdot (LJ)^2 = LK(LK \cdot LK)$	5. Distributive property

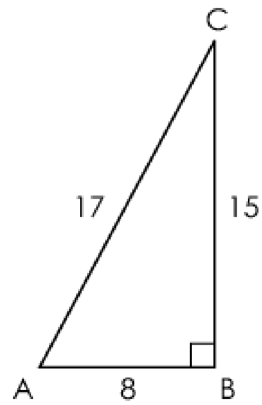
Geometry
Spring 2018 Item Release

Question 8

Question and Scoring Guidelines

Question 8

A right triangle ABC is shown.



What is cos A?

Points Possible: 1

Content Cluster: Define trigonometric ratios and solve problems involving right triangles.

Content Standard: Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles. *(G.SRT.6)*

Scoring Guidelines

Exemplar Response

- $\frac{8}{17}$

Other Correct Responses

- Any equivalent value
- Any equivalent value between 0.47 and 0.471, inclusive

For this item, a full-credit response includes:

- A correct value (1 point).

Geometry
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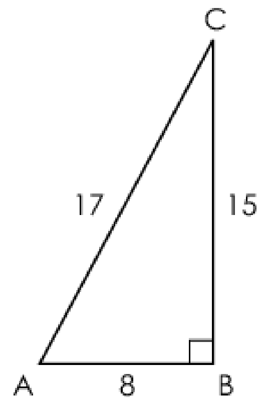
Question 8

Sample Responses

Sample Response: 1 point

A right triangle ABC is shown.



What is cos A?

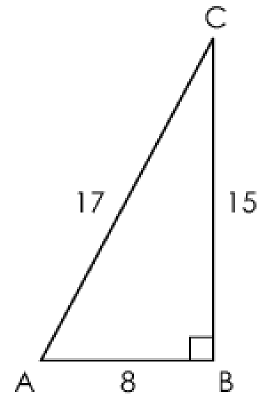
0.47

Notes on Scoring

This response earns full credit (1 point) because it shows the correct value of cos A. For right triangles, the cosine of an acute angle is the ratio of the lengths of the adjacent leg to the hypotenuse. Based on this definition, cos A = $\frac{AB}{AC}$, or $\frac{8}{17}$. Any value between 0.47 and 0.471 is also accepted.

Sample Response: 1 point

A right triangle ABC is shown.



What is cos A?

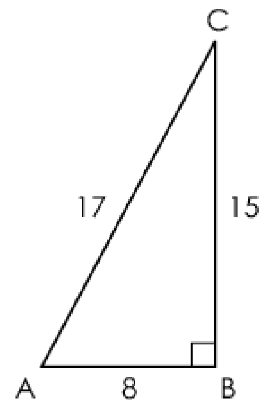
0.471

Notes on Scoring

This response earns full credit (1 point) because it shows the correct value of cos A. For right triangles, the cosine of an acute angle is the ratio of the lengths of the adjacent leg to the hypotenuse. Based on this definition, cos A = $\frac{AB}{AC}$, or $\frac{8}{17}$. Any value between 0.47 and 0.471 is also accepted.

Sample Response: 0 points

A right triangle ABC is shown.



What is cos A?

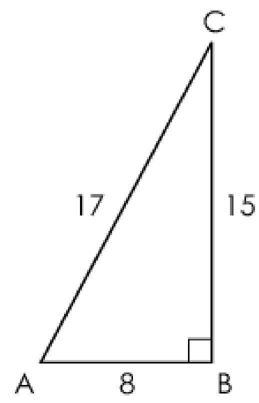
0.88

Notes on Scoring

This response earns no credit (0 points) because it shows an incorrect value for cos A. The student may confuse the definition of cos A with the definition of sin A. In right triangles, the sine of an acute angle is the ratio of the lengths of the opposite leg to the hypotenuse. For this triangle, sin A = $\frac{15}{17}$, or approximately 0.88.

Sample Response: 0 points

A right triangle ABC is shown.



What is cos A?

2.125

Notes on Scoring

This response earns no credit (0 points) because it shows an incorrect value for cos A. The student may reciprocate the definition of the cosine of an acute angle, and calculates it as the ratio of the lengths of the hypotenuse, 17, to the adjacent leg, 8, or 2.125 instead of the ratio of the lengths of the adjacent leg to the hypotenuse.

Geometry
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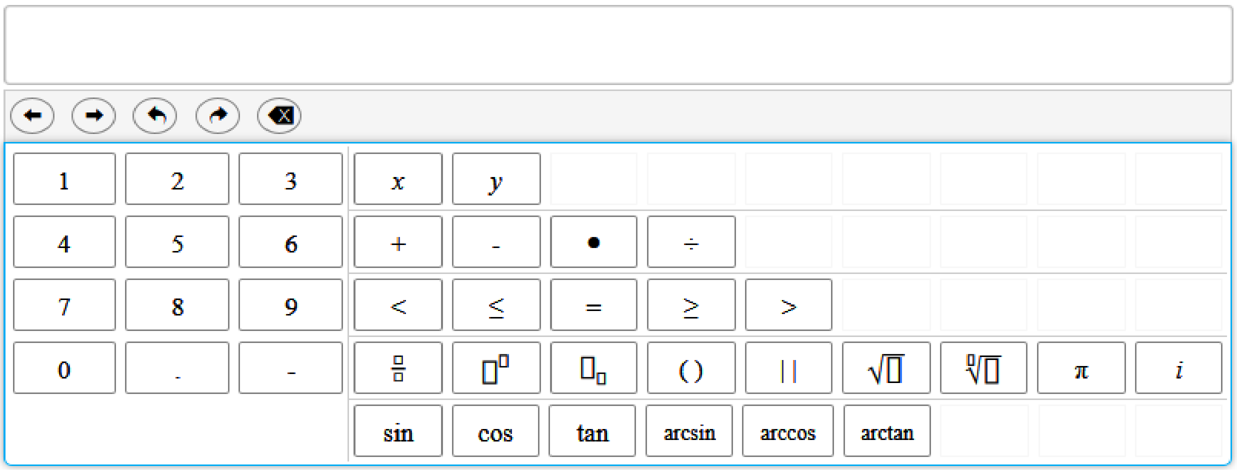
Question 9

Question and Scoring Guidelines

Question 9

Line k has a slope of -5. Line j is perpendicular to line k and passes through the point $(5, 9)$.

Create the equation for line j.



Points Possible: 1

Content Cluster: Use coordinates to prove simple geometric theorems algebraically.

Content Standard: Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point). *(G.GPE.5)*

Scoring Guidelines

Exemplar Response

- $y = \frac{1}{5}x + 8$

Other Correct Responses

- Any equivalent value

For this item, a full-credit response includes:

- A correct equation (1 point).

Geometry
Spring 2018 Item Release

Question 9

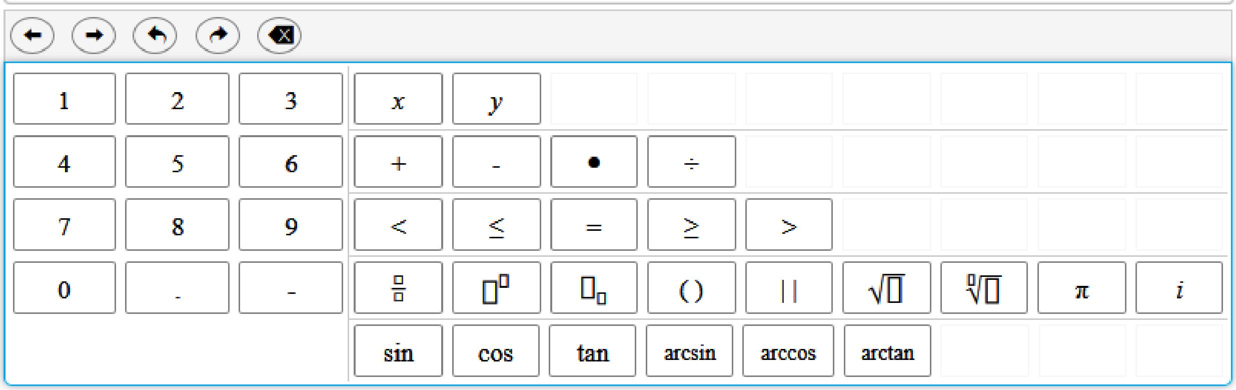
Sample Responses

Sample Response: 1 point

Line k has a slope of −5. Line j is perpendicular to line k and passes through the point (5, 9).

Create the equation for line j.

$$y=\frac{1}{5}x+8$$



Notes on Scoring

This response earns full credit (1 point) because it shows a correct equation of line *j* that is perpendicular to line *k*. It is given that the slope of line *k* is –5. Therefore, the slope of line *j* is $\frac{1}{5}$ because the slope of a perpendicular line is the opposite reciprocal, excluding the case of vertical and horizontal lines.

There are several ways to obtain the equation of line *j*. One way is to use the point-slope equation of a line $y - y_1 = m(x - x_1)$, where m is the slope of the line and (x_1, y_1) is a point on the line. Since line *j* passes through the point (5, 9) and has a slope of $\frac{1}{5}$, an equation for line *j* is $y - 9 = \frac{1}{5}(x - 5)$.

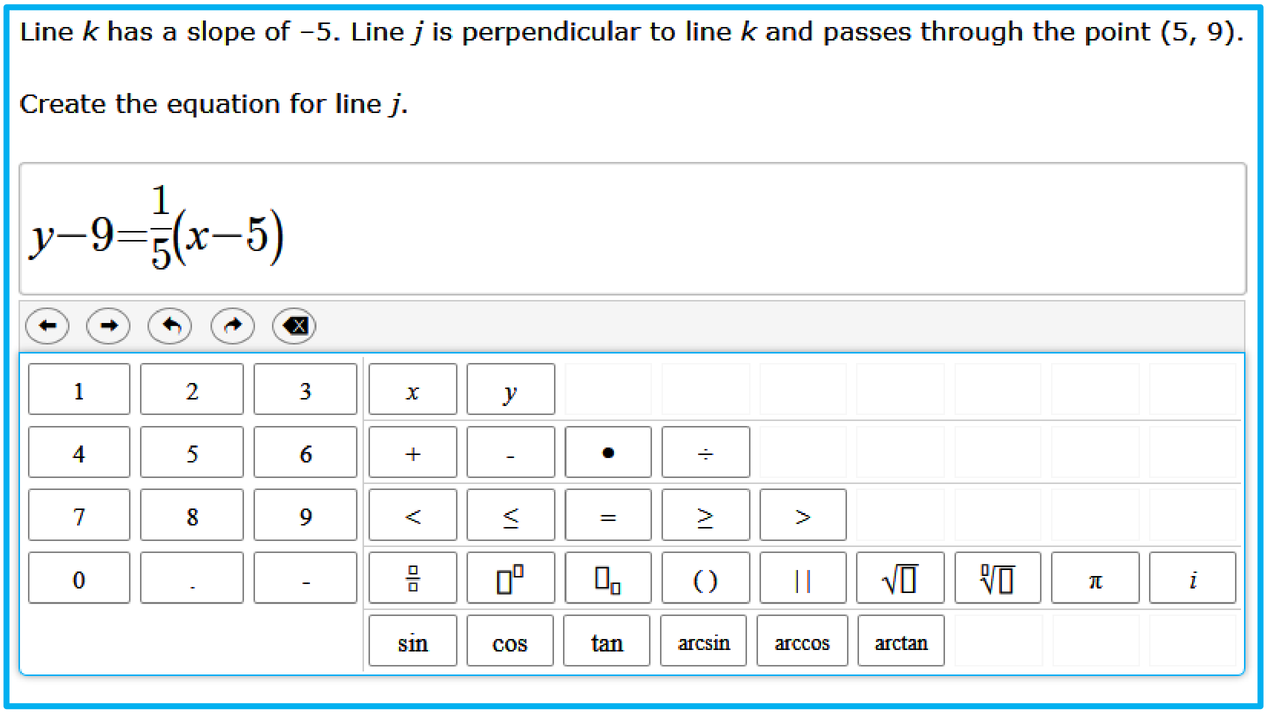
This equation can be restated in slope-intercept form, $y = mx + b$, by applying the distributive property on the right side of the point-slope equation, and then adding 9 to both sides.

$$y - 9 = \frac{1}{5}(x - 5)$$

$$y - 9 = \frac{1}{5}x - 1$$

$$y = \frac{1}{5}x + 8$$

Sample Response: 1 point



Notes on Scoring

This response earns full credit (1 point) because it shows a correct equation of line *j* that is perpendicular to line *k*. It is given that the slope of line *k* is –5. Therefore, the slope of line *j* is $\frac{1}{5}$ because the slope of a perpendicular line is the opposite reciprocal, excluding the case of vertical and horizontal lines.

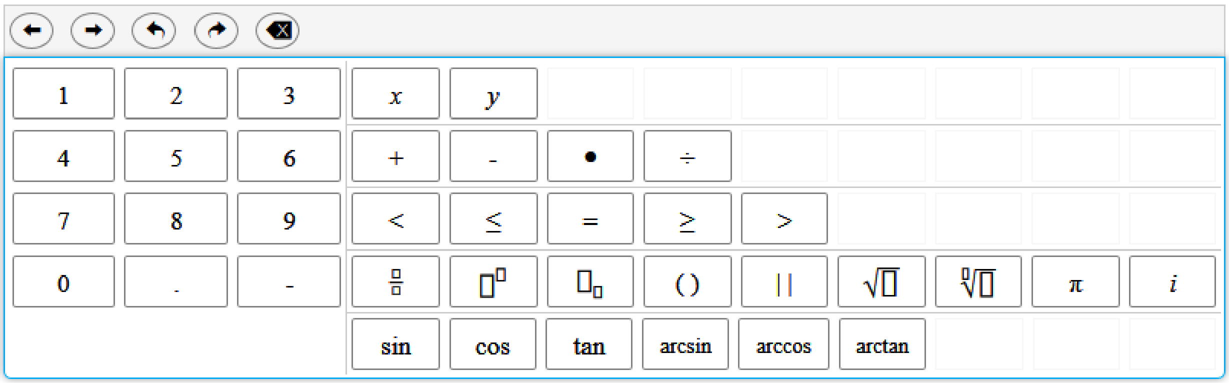
There are several ways to obtain the equation of the line *j*. One way is to use the point-slope equation of a line $y - y_1 = m(x - x_1)$, where m is the slope of the line and (x_1, y_1) is a point on the line. Since line *j* passes through the point (5, 9) and has a slope of $\frac{1}{5}$, the equation for line *j* is $y - 9 = \frac{1}{5}(x - 5)$.

Sample Response: 0 points

Line *k* has a slope of −5. Line *j* is perpendicular to line *k* and passes through the point (5, 9).

Create the equation for line *j*.

$$y-9=-5(x-5)$$



Notes on Scoring

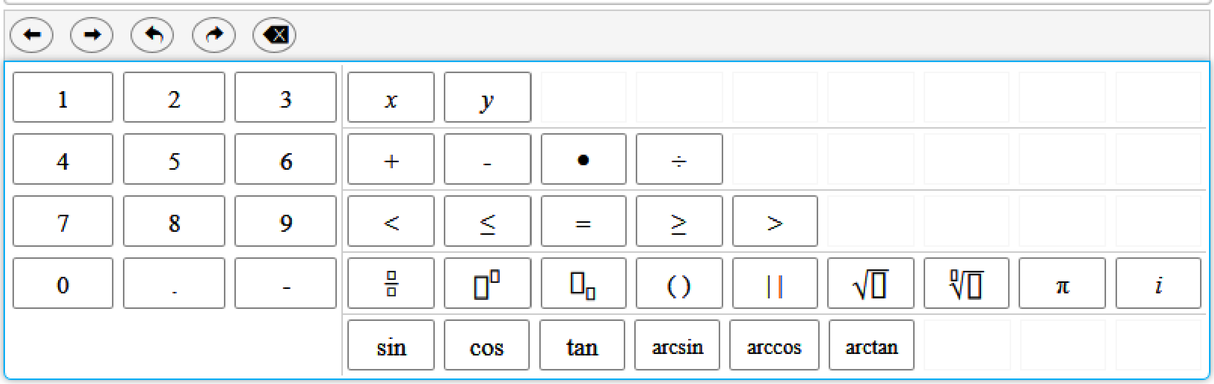
This response earns no credit (0 points) because it shows an incorrect equation for line *j*. The student may confuse the perpendicular and parallel lines, thinking that perpendicular lines have equal slopes. He or she may use the point-slope equation of a line, $y - y_1 = m(x - x_1)$, to substitute the point (5, 9) for (x_1, y_1) and –5 for the slope m, so that the equation for line *j* becomes $y - 9 = -5(x - 5)$.

Sample Response: 0 points

Line k has a slope of -5. Line j is perpendicular to line k and passes through the point (5, 9).

Create the equation for line j.

$$y=-\frac{1}{5}x+10$$



Notes on Scoring

This response earns no credit (0 points) because it shows an incorrect equation for line j. The student may think that the slopes of perpendicular lines are reciprocals instead of opposite reciprocals. The student then may substitute the point (5, 9) for (x_1, y_1) and $-\frac{1}{5}$ for the slope m into the point-slope equation of a line, $y - y_1 = m(x - x_1)$, so that the equation for line j becomes $y - 9 = -\frac{1}{5}(x - 5)$. The student then may rewrite the equation to be in slope-intercept form, $y = mx + b$, which results in the equation $y = -\frac{1}{5}x + 10$.

Geometry
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Question 11

Question and Scoring Guidelines

Question 11

Jeremy wants to know the density of a rock in grams per cubic centimeter. The rock has a mass of 1.08 kilograms and a volume of 400 cubic centimeters.

What is the density of the rock, in **grams** per cubic centimeter $\left(\frac{g}{cm^3}\right)$?

$\frac{g}{cm^3}$

1	2	3
4	5	6
7	8	9
	0	
.	-	$\frac{\square}{\square}$

Points Possible: 1

Content Cluster: Apply geometric concepts in modeling situations.

Content Standard: Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot). *(G.MG.2)*

Scoring Guidelines

Exemplar Response

- $2.7\ \frac{\mathrm{g}}{\mathrm{cm}^3}$

Other Correct Responses

- Any equivalent value

For this item, a full-credit response includes:

- The correct density (1 point).

Geometry
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Question 11

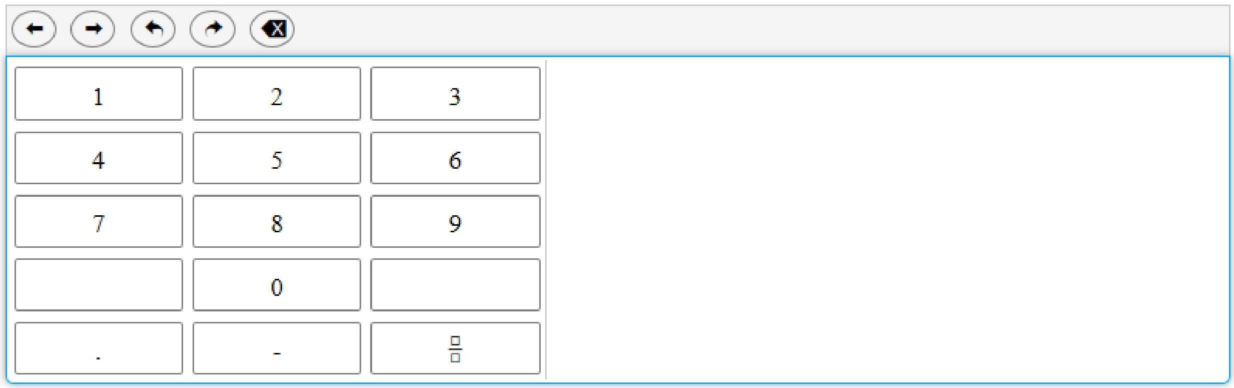
Sample Responses

Sample Response: 1 point

Jeremy wants to know the density of a rock in grams per cubic centimeter. The rock has a mass of 1.08 kilograms and a volume of 400 cubic centimeters.

What is the density of the rock, in **grams** per cubic centimeter $\left(\frac{g}{cm^3}\right)$?

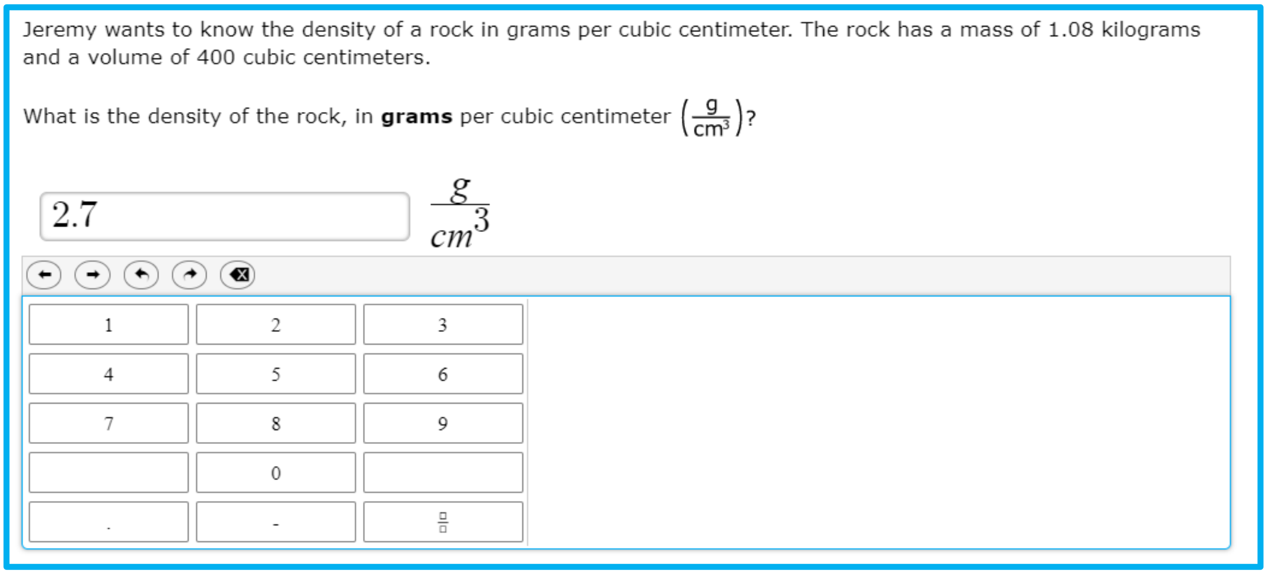
$$\frac{1080}{400} \quad \frac{g}{cm^3}$$



Notes on Scoring

This response earns full credit (1 point) because it shows an exact answer for the density of the rock in g/cm^3. Density is a measure that expresses the quantity of substance per unit of volume, area, or length. In this situation, the density of the rock, in grams per cubic centimeter, is the ratio of 1080 g (the rock’s mass of 1.08 kg converted to grams) to the rock’s volume of 400 cm^3, or $\frac{1080}{400}$ g/cm^3.

Sample Response: 1 point



Jeremy wants to know the density of a rock in grams per cubic centimeter. The rock has a mass of 1.08 kilograms and a volume of 400 cubic centimeters.

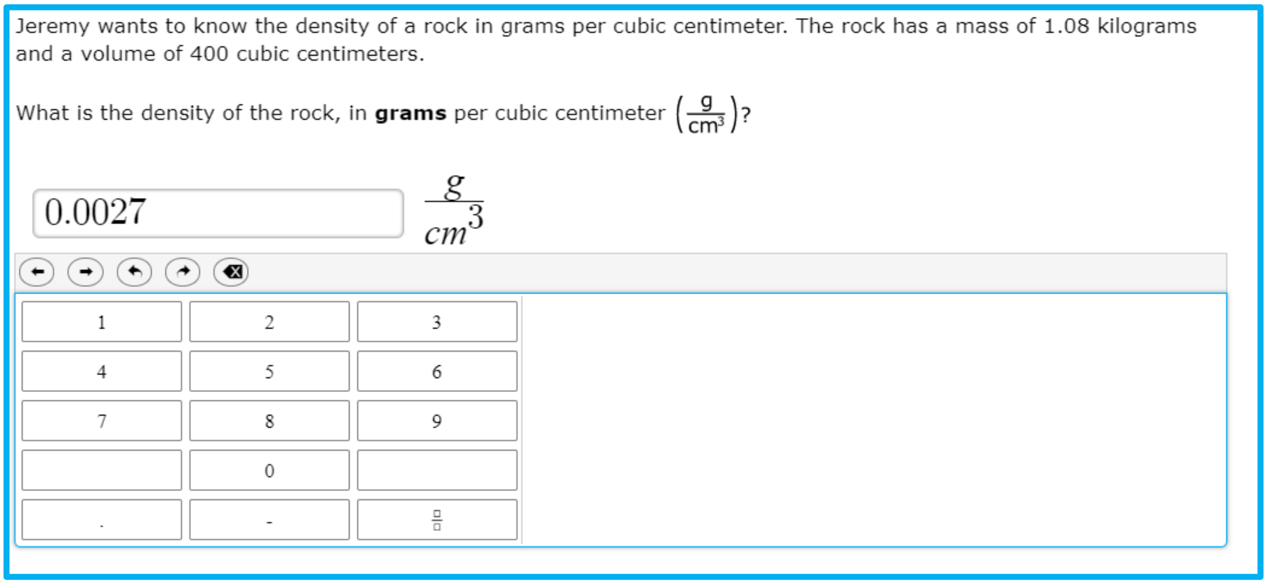
What is the density of the rock, in **grams** per cubic centimeter $\left(\frac{g}{cm^3}\right)$?

2.7 $\frac{g}{cm^3}$

Notes on Scoring

This response earns full credit (1 point) because it shows an exact answer for the density of the rock in g/cm^3. Density is a measure that expresses the quantity of substance per unit of volume, area, or length. In this situation, the density of the rock, in grams per cubic centimeter, is the ratio of 1080 g (the rock's mass of 1.08 kg converted to grams) to the rock's volume of 400 g/cm^3, or $\frac{1080}{400}$ g/cm^3 = 2.7 g/cm^3.

Sample Response: 0 points



Jeremy wants to know the density of a rock in grams per cubic centimeter. The rock has a mass of 1.08 kilograms and a volume of 400 cubic centimeters.

What is the density of the rock, in **grams** per cubic centimeter $\left(\frac{g}{cm^3}\right)$?

0.0027 $\frac{g}{cm^3}$

> **Notes on Scoring**
>
> This response earns no credit (0 points) because it shows an incorrect answer for the density of the rock, in g/cm^3. The student may not convert 1.08 kg to grams and calculates the density in kg/cm^3 instead of in g/cm^3.

Sample Response: 0 points

Jeremy wants to know the density of a rock in grams per cubic centimeter. The rock has a mass of 1.08 kilograms and a volume of 400 cubic centimeters.

What is the density of the rock, in **grams** per cubic centimeter $\left(\frac{g}{cm^3}\right)$?

$\frac{10}{27}$ $\frac{g}{cm^3}$

1	2	3
4	5	6
7	8	9
	0	
.	-	$\frac{\square}{\square}$

Notes on Scoring

This response earns no credit (0 points) because it shows an incorrect answer for the density of the rock, in g/cm³. The student may switch the parts of the ratio in the density definition and divides the volume, 400, by the quantity,1080, instead of 1080 by 400.

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Question 12

Question and Scoring Guidelines

Question 12

The two-way table shows the number of births, in thousands, in the United States for the years 2010 and 2011.

	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.	Total
2010	324	303	340	327	325	338	346	359	350	342	337	326	4017
2011	322	299	330	315	328	335	348	362	346	331	328	322	3966

A baby born in 2011 is randomly selected.

What is the probability that the baby was born in February?

Points Possible: 1

Content Cluster: Use the rules of probability to compute probabilities of compound events in a uniform probability model.

Content Standard: Find the conditional probability of *A* given *B* as the fraction of *B*'s outcomes that also belong to *A*, and interpret the answer in terms of the model. *(S.CP.6)*

Scoring Guidelines

Exemplar Response

- $\frac{299}{3966}$

Other Correct Responses

- Any equivalent value
- Any decimal between 0.075 and 0.0754

For this item, a full-credit response includes:

- A correct response (1 point).

Geometry
Spring 2018 Item Release

Question 12

Sample Responses

Sample Response: 1 point

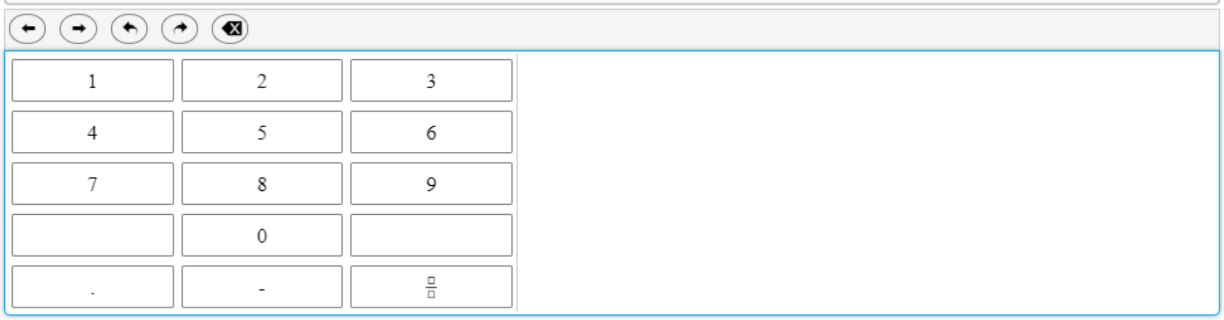
The two-way table shows the number of births, in thousands, in the United States for the years 2010 and 2011.

	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.	Total
2010	324	303	340	327	325	338	346	359	350	342	337	326	4017
2011	322	299	330	315	328	335	348	362	346	331	328	322	3966

A baby born in 2011 is randomly selected.

What is the probability that the baby was born in February?

$$\frac{299}{3966}$$



Notes on Scoring

This response earns full credit (1 point) because it shows the correct probability for a baby being born in February 2011. To calculate the correct probability, it may be helpful to reword the question as, “What is the probability that the baby was born in February, given that the baby was born in 2011?”. Since the total number of babies who were born in 2011 was 3966, and 299 of those babies were born in February, the probability that the baby was born in February 2011 is $\frac{299}{3966}$.

Sample Response: 1 point

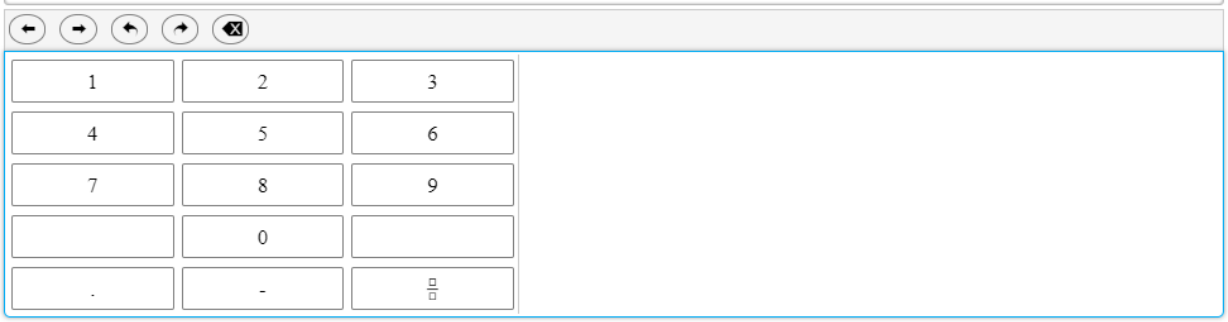
The two-way table shows the number of births, in thousands, in the United States for the years 2010 and 2011.

	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.	Total
2010	324	303	340	327	325	338	346	359	350	342	337	326	4017
2011	322	299	330	315	328	335	348	362	346	331	328	322	3966

A baby born in 2011 is randomly selected.

What is the probability that the baby was born in February?

0.075



Notes on Scoring

This response earns full credit (1 point) because it shows the correct decimal approximation, 0.075, of the probability 299/3966 for a baby being born in February 2011.

Sample Response: 0 points

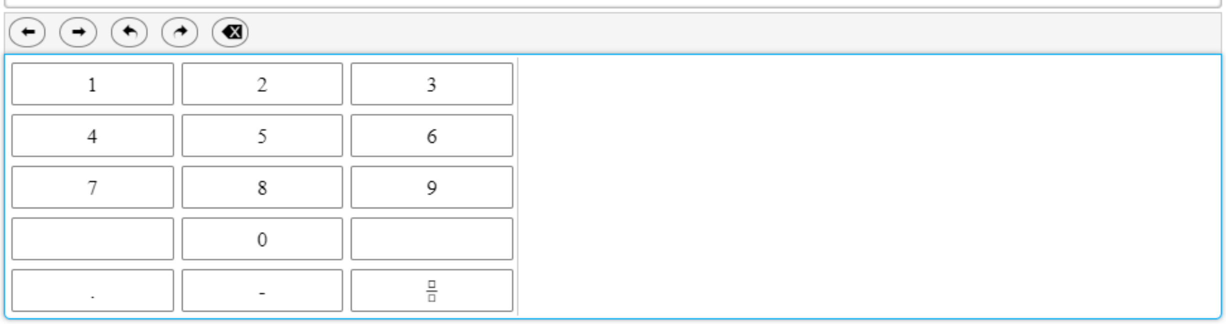
The two-way table shows the number of births, in thousands, in the United States for the years 2010 and 2011.

	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.	Total
2010	324	303	340	327	325	338	346	359	350	342	337	326	4017
2011	322	299	330	315	328	335	348	362	346	331	328	322	3966

A baby born in 2011 is randomly selected.

What is the probability that the baby was born in February?

$$\frac{303}{4017}$$



Notes on Scoring

This response earns no credit (0 points) because it shows an incorrect probability for the baby being born in February 2011. The student may view the wrong row of the table and mistakenly calculates the probability of the baby being born in February 2010.

Sample Response: 0 points

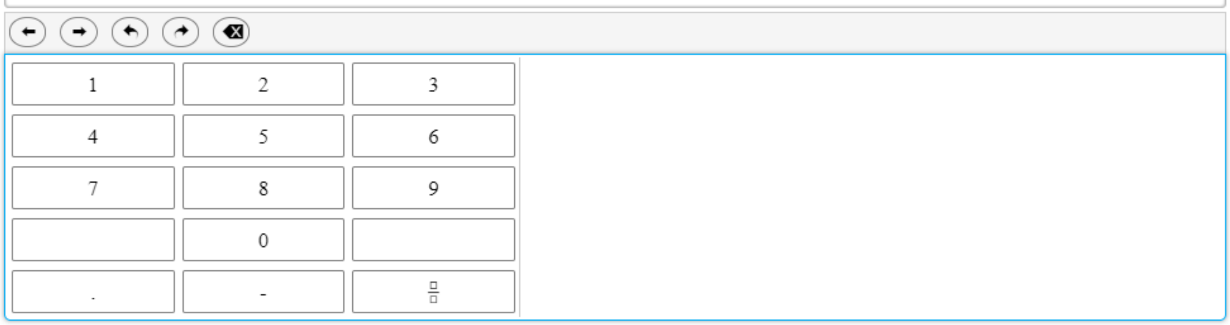
The two-way table shows the number of births, in thousands, in the United States for the years 2010 and 2011.

	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.	Total
2010	324	303	340	327	325	338	346	359	350	342	337	326	4017
2011	322	299	330	315	328	335	348	362	346	331	328	322	3966

A baby born in 2011 is randomly selected.

What is the probability that the baby was born in February?

299000



Notes on Scoring

This response earns no credit (0 points) because it shows an incorrect probability for the baby being born in February 2011. The student may provide the number of babies, 299000, born in February 2011 instead of the probability for the randomly selected baby being born in February 2011.

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Question 15

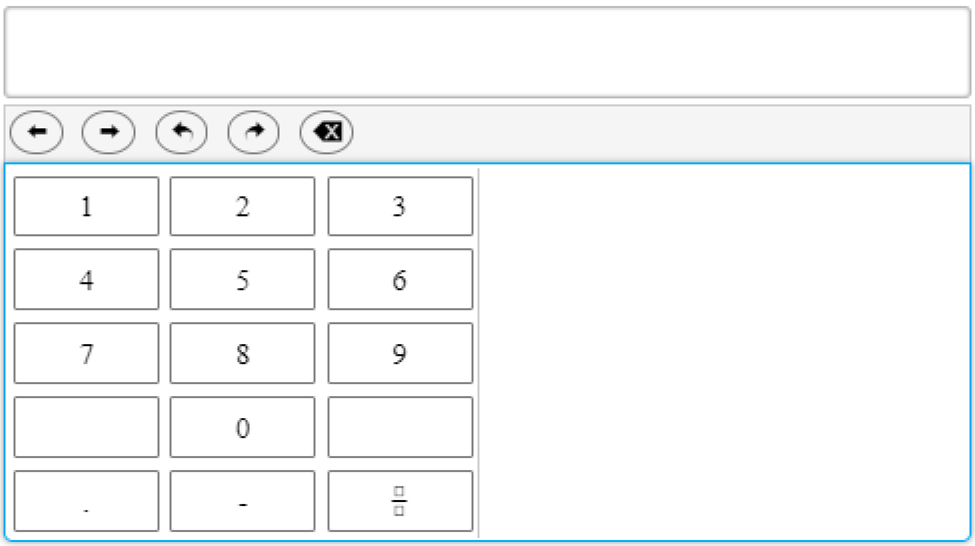
Question and Scoring Guidelines

Question 15

A total of 50 students play either soccer or lacrosse.

- 20 girls play lacrosse.
- 20 boys play either soccer or lacrosse.
- 20 students play soccer.

What is the probability that a student plays soccer or is a girl?



Points Possible: 1

Content Cluster: Use the rules of probability to compute probabilities of compound events in a uniform probability model.

Content Standard: Apply the Addition Rule, $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$, and interpret the answer in terms of the model. *(S.CP.7)*

Scoring Guidelines

Exemplar Response

- 0.8

Other Correct Responses

- Any equivalent value

For this item, a full-credit response includes:

- A correct probability (1 point).

Geometry
Spring 2018 Item Release

Question 15

Sample Responses

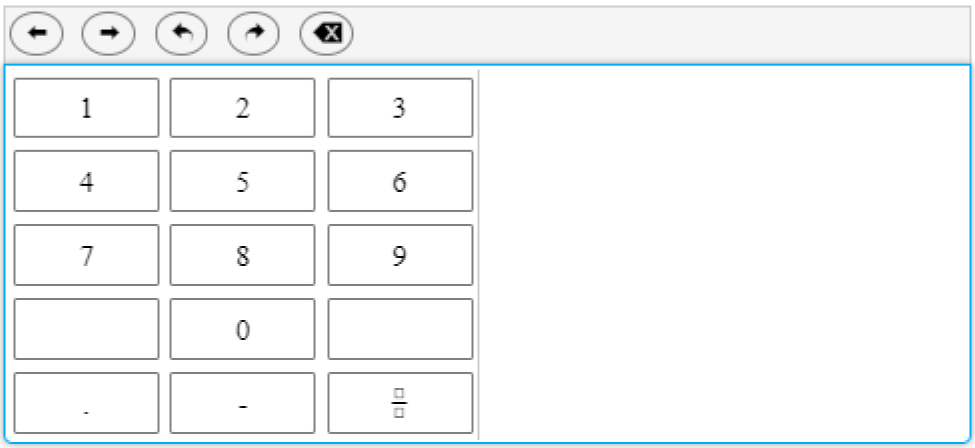
Sample Response: 1 point

A total of 50 students play either soccer or lacrosse.

- 20 girls play lacrosse.
- 20 boys play either soccer or lacrosse.
- 20 students play soccer.

What is the probability that a student plays soccer or is a girl?

0.8



Notes on Scoring

This response earns full credit (1 point) because it shows the correct probability that a student plays soccer or is a girl. One way to calculate this probability is to represent the situation by a two-way frequency table and then use the entries from the table and the Addition Rule for the probability of compound events.

Following from the given information (bold in the table), the total number of students playing either soccer or lacrosse is 50 (bottom right cell), the frequency of "boys playing soccer or lacrosse" is 20 (row 1, column 3) the frequency of "students playing soccer" is 20 (row 3, column 1), and the frequency of "girls playing lacrosse" is 20 (row 2, column 2).
The next step is to calculate the frequency of "Students that are girls" as 50 – 20 = 30 (row 2, column 3) and the frequency of "Students that are girls and play soccer" as 30 – 20 = 10 (row 2, column 1).

	Soccer	Lacrosse	Total
Boys	10	10	**20**
Girls	10 **(A and B)**	**20**	30 **(B)**
Total	**20 (A)**	30	**50**

Next, the probability that a student plays soccer OR is a girl can be calculated using the Addition Rule for the probability of compound events, P(A or B) = P(A) + P(B) – P(A and B), where

P(A) is the probability that a student plays soccer $\left(\frac{\text{row 3, column 1}}{50}\right)$

P(B) is the probability that a student is a girl $\left(\frac{\text{row 2, column 3}}{50}\right)$

P (A and B) is the probability that a student plays soccer and is a girl $\left(\frac{\text{row 2, column 1}}{50}\right)$

These values are substituted into the Addition Rule formula to complete the calculation of P(A or B).

$$P(A \text{ or } B) = \frac{20}{50} + \frac{30}{50} - \frac{10}{50} = \frac{40}{50}$$

$$= 0.8$$

Sample Response: 1 point

A total of 50 students play either soccer or lacrosse.

- 20 girls play lacrosse.
- 20 boys play either soccer or lacrosse.
- 20 students play soccer.

What is the probability that a student plays soccer or is a girl?

$$\frac{4}{5}$$

Notes on Scoring

This response earns full credit (1 point) because it shows a correct probability that a student plays soccer or is a girl in fractional form, $\frac{4}{5}$, equivalent to the decimal form, 0.8.

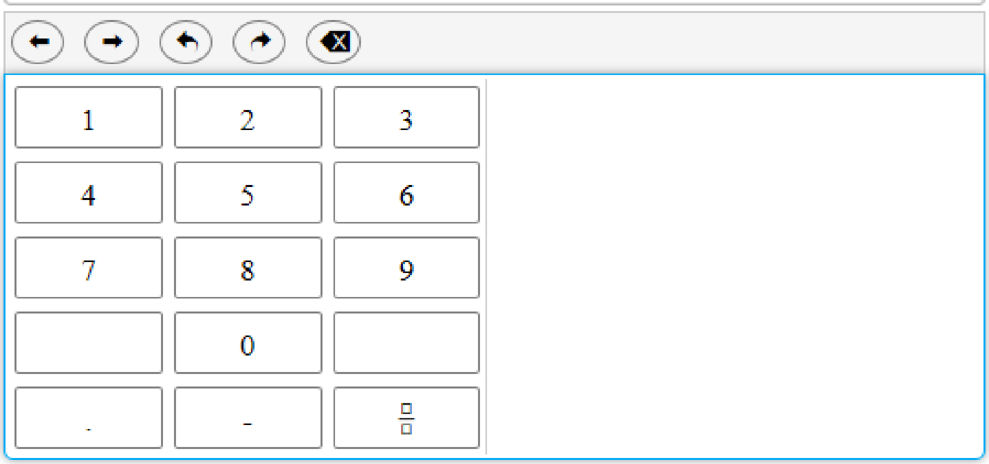
Sample Response: 0 points

A total of 50 students play either soccer or lacrosse.

- 20 girls play lacrosse.
- 20 boys play either soccer or lacrosse.
- 20 students play soccer.

What is the probability that a student plays soccer or is a girl?

$$\frac{30}{50}$$



Notes on Scoring

This response earns no credit (0 points) because it shows an incorrect probability that a student plays soccer or is a girl. The student may calculate the probability of randomly choosing a girl, 30, out of a total of 50 students.

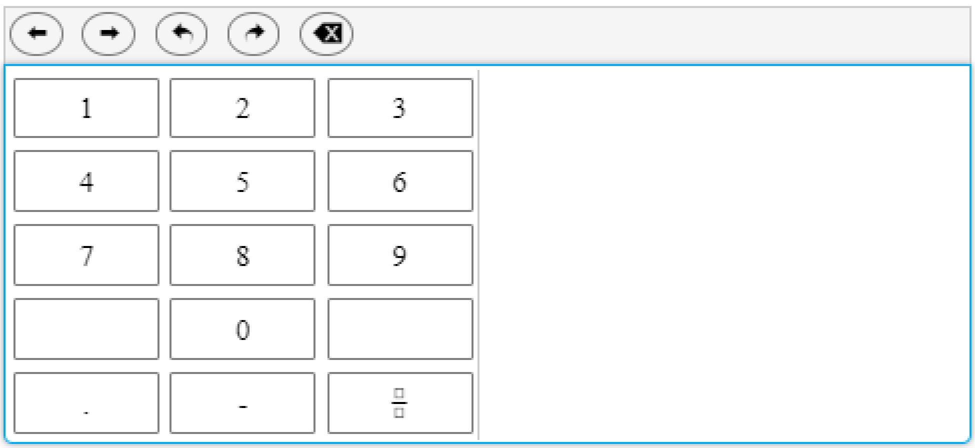
Sample Response: 0 points

A total of 50 students play either soccer or lacrosse.

- 20 girls play lacrosse.
- 20 boys play either soccer or lacrosse.
- 20 students play soccer.

What is the probability that a student plays soccer or is a girl?

0.4



Notes on Scoring

This response earns no credit (0 points) because it shows an incorrect probability that a student plays soccer or is a girl. The student may calculate the probability of randomly choosing a student who play soccer, 20, out of a total of 50 students, or $\frac{20}{50}$, which is 0.4 in decimal form.

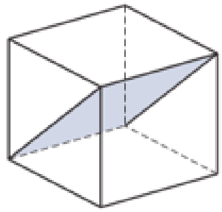
Geometry
Spring 2018 Item Release

Question 16

Question and Scoring Guidelines

Question 16

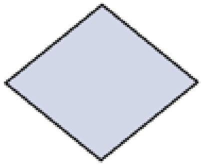
A cube is sliced as shown.



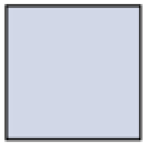
What is the shape of the cross section?

Ⓐ Rectangle

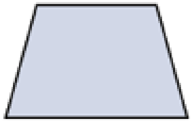
Ⓑ Rhombus



Ⓒ Square



Ⓓ Trapezoid



Points Possible: 1

Content Cluster: Visualize relationships between two-dimensional and three-dimensional objects.

Content Standard: Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects. *(G.GMD.4)*

Scoring Guidelines

Rationale for Option A: **Key** – The student notes that the cross section would be a parallelogram with two pairs of congruent sides. Since all edges of a cube are congruent, the two sides of a cross section are congruent and parallel. The remaining two sides of the cross section are also congruent and parallel because they are the diagonals of parallel faces of the cube. The student also notices that the cross section is not a square because the adjacent sides of the cross section are not congruent (diagonal of a square face of the cube is not congruent to the edges of the cube).

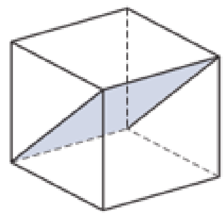
Rationale for Option B: The student may think that the cross section is a rhombus but fails to notice that not all four sides are congruent. The adjacent sides of the cross section are not congruent.

Rationale for Option C: The student may think that the cross section is a square but fails to notice that the adjacent sides are not congruent. One pair of opposite sides of a cross section are the diagonals of square faces, and they are not congruent to the edges of the cube.

Rationale for Option D: The student may think that the cross section is a trapezoid – in this case a quadrilateral with one pair of non-parallel opposite sides. However, since the cross section is formed by the diagonals of parallel square faces of the cube and two parallel edges of the cube, both pairs of opposite sides of the cross section must be parallel.

Sample Response: 1 point

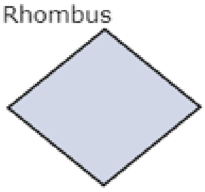
A cube is sliced as shown.



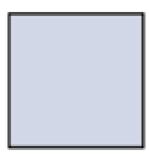
What is the shape of the cross section?

● Rectangle

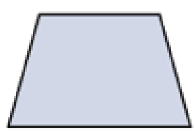
Ⓑ Rhombus



Ⓒ Square



Ⓓ Trapezoid



Geometry
Spring 2018 Item Release

Question 18

Question and Scoring Guidelines

Question 18

Events A and B are independent.

P(A and B) = 0.25

Enter possible probabilities for events A and B.

$P(A) =$ ______

$P(B) =$ ______

Points Possible: 1

Content Cluster: Understand independence and conditional probability and use them to interpret data.

Content Standard: Understand that two events *A* and *B* are independent if the probability of *A* and *B* occurring together is the product of their probabilities, and use this characterization to determine if they are independent. *(S.CP.2)*

Scoring Guidelines

Exemplar Response

- P(A) = 0.5
  P(B) = 0.5

Other Correct Responses

- Any values such that *P(A) x P(B)* = 0.25 and both values are greater than 0 and less than or equal to 1

For this item, a full-credit response includes:

- Two correct probabilities (1 point).

Geometry
Spring 2018 Item Release

Question 18

Sample Responses

Sample Response: 1 point

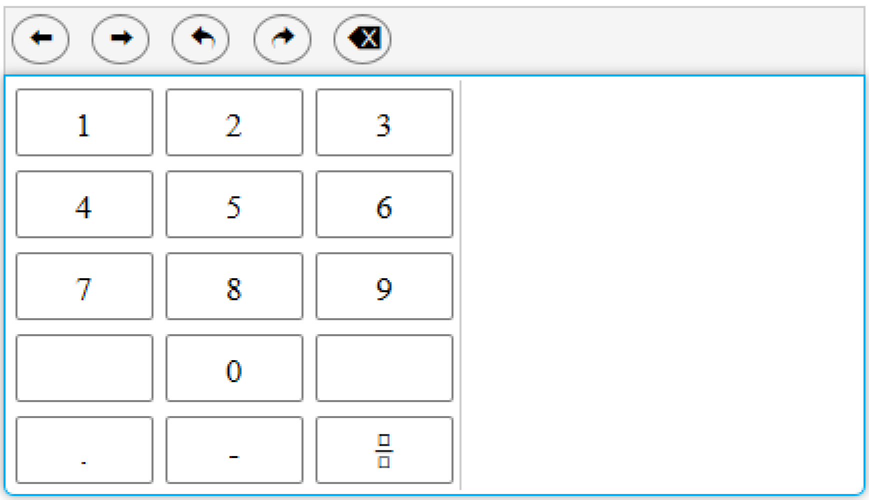
Events A and B are independent.

P(A and B) = 0.25

Enter possible probabilities for events A and B.

$P(A) =$ 0.4

$P(B) =$ 0.625



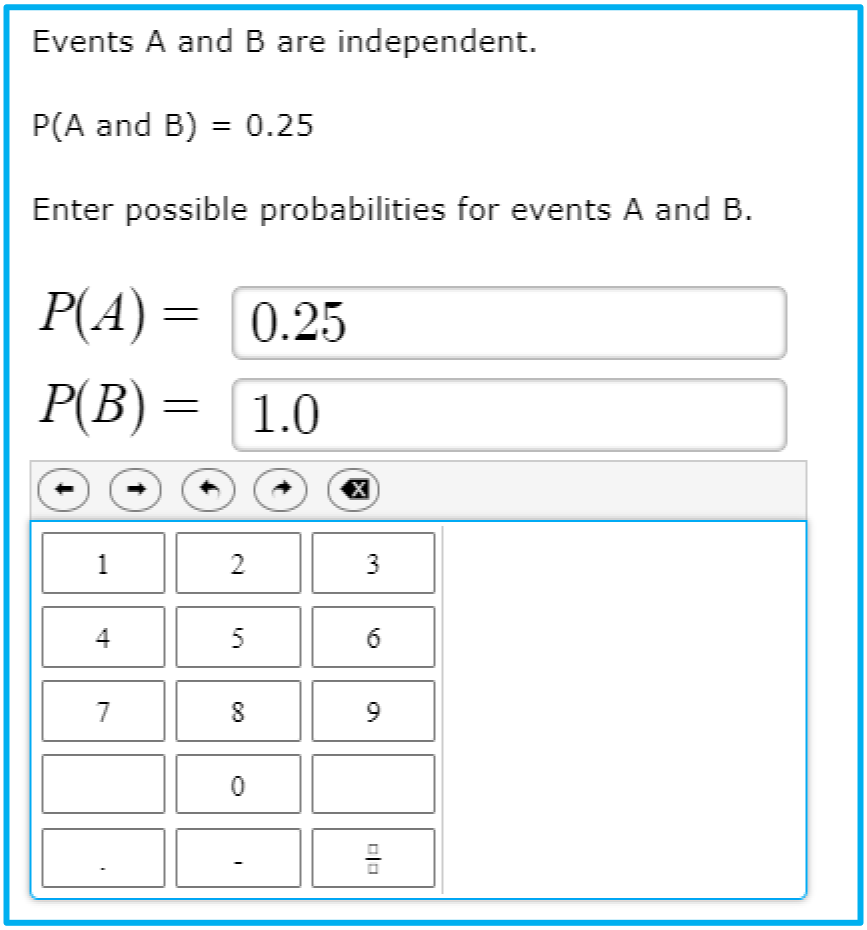
Notes on Scoring

This response earns full credit (1 point) because it shows two correct possible probabilities for the independent events A and B.

For two independent events A and B, the probability of the intersection of the events P(A and B) can be calculated using the equation P(A and B) = P(A) • P(B). Overall, the probability of an event occurring is greater than or equal to 0 and less than or equal to 1.

In this open-ended situation, since the product of two probabilities is 0.25, each individual probability must be a value that is positive and less than or equal to 1. Therefore, any two values that are positive and less than or equal to 1 whose product is 0.25 could be the probabilities of these two independent events. For example, P(A) can be 0.4 and P(B) can be 0.625 because 0.4 • 0.625 = 0.25.

Sample Response: 1 point

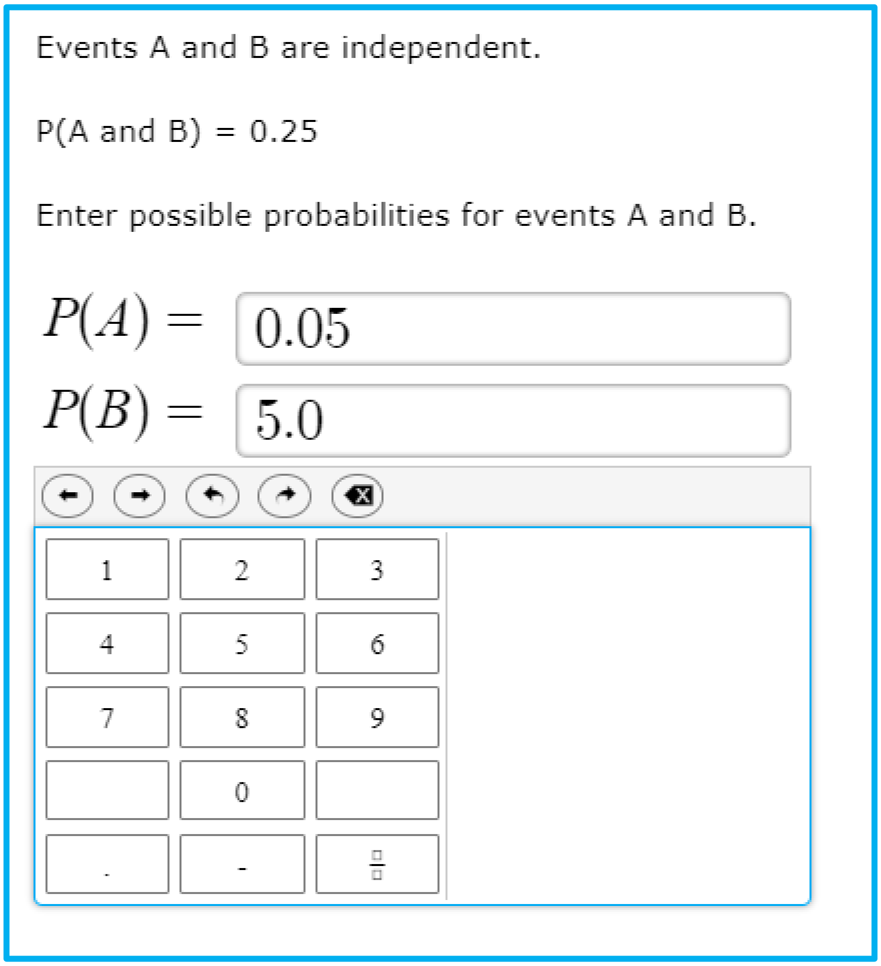


Notes on Scoring

This response earns full credit (1 point) because it shows two correct possible probabilities for the independent events A and B.

Since the product of the two probabilities is 0.25, each individual probability must be a value that is positive and less than or equal to 1. Therefore, two positive values such as 0.25 and 1, whose product is 0.25 are possible correct probabilities for the independent events A and B.

Sample Response: 0 points


Events A and B are independent.

P(A and B) = 0.25

Enter possible probabilities for events A and B.

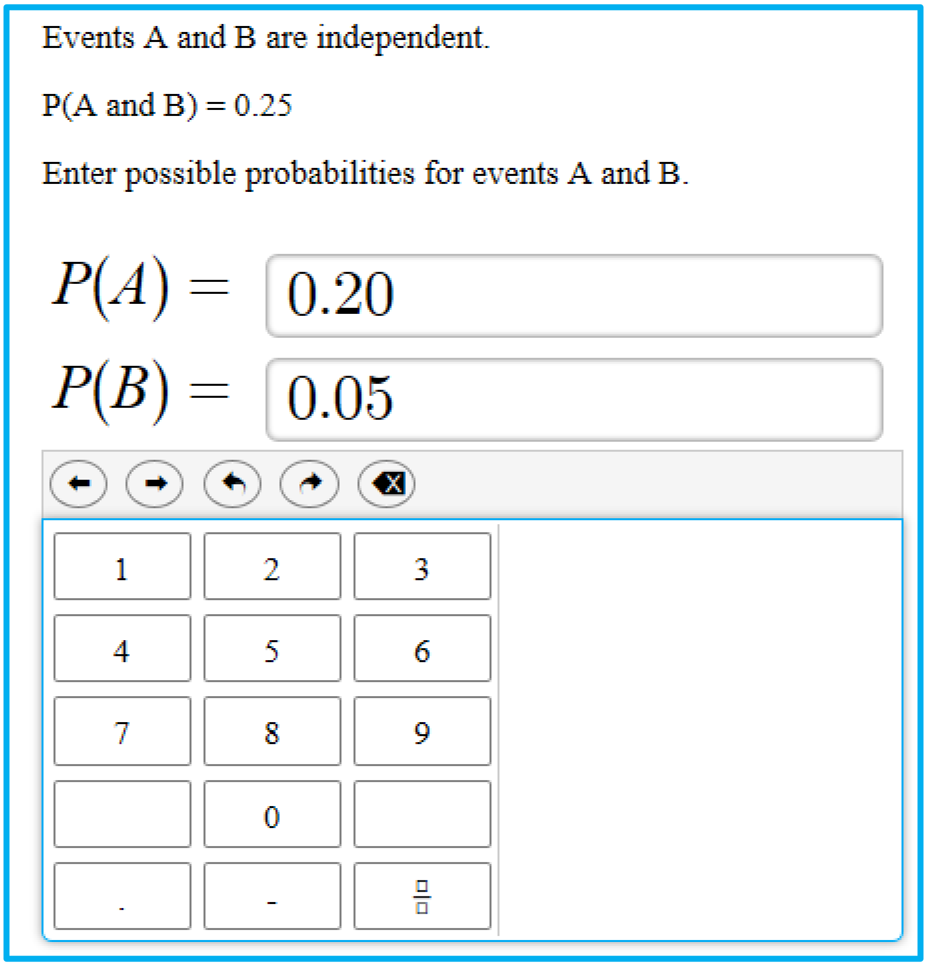
$P(A) =$ 0.05

$P(B) =$ 5.0

Notes on Scoring

This response earns no credit (0 points) because it shows one impossible probability, P(B) = 5. Since the probability of an event occurring is always greater than or equal to 0 and less than or equal to 1, the entire response is incorrect, even though P(A) • P(B) = 0.25.

Sample Response: 0 points



Notes on Scoring

This response earns no credit (0 points) because the product of P(A) and P(B) is not 0.25. The student may think that when two events are independent, the probability that both events occur is the sum of their individual probabilities instead of the product of their individual probabilities.

Geometry
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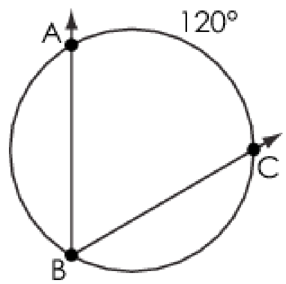
Question 19

Question and Scoring Guidelines

Question 19

Angle ABC is inscribed in a circle as shown.



What is the measure, in degrees, of $\angle ABC$?

Points Possible: 1

Content Cluster: Understand and apply theorems about circles.

Content Standard: Identify and describe relationships among inscribed angles, radii, and chords. *Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle. (G.C.2)*

Scoring Guidelines

<u>Exemplar Response</u>

- 60 degrees

<u>Other Correct Responses</u>

- Any equivalent value

For this item, a full-credit response includes:

- A correct angle measure (1 point).

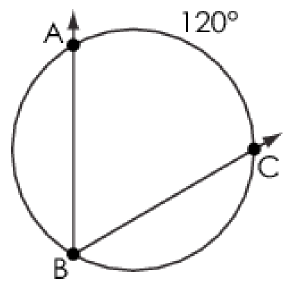
Geometry
Spring 2018 Item Release

Question 19

Sample Responses

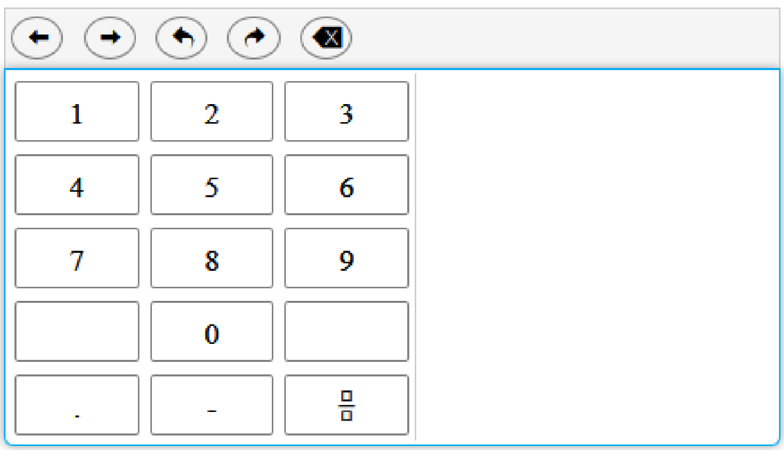
Sample Response: 1 point

Angle ABC is inscribed in a circle as shown.



What is the measure, in degrees, of ∠ABC?

60 *degrees*

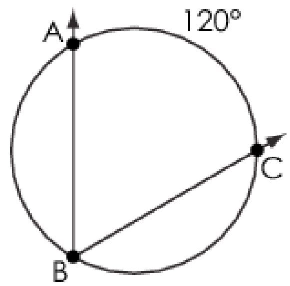


Notes on Scoring

This response earns full credit (1 point) because it shows the correct measure of the inscribed angle.
According to the Inscribed Angle Theorem, the measure of an angle inscribed in a circle is one half the measure of its intercepted arc. In this situation, since the measure of the intercepted arc is 120º, the measure of the inscribed angle ABC is $\frac{1}{2} \bullet 120° = 60°$.

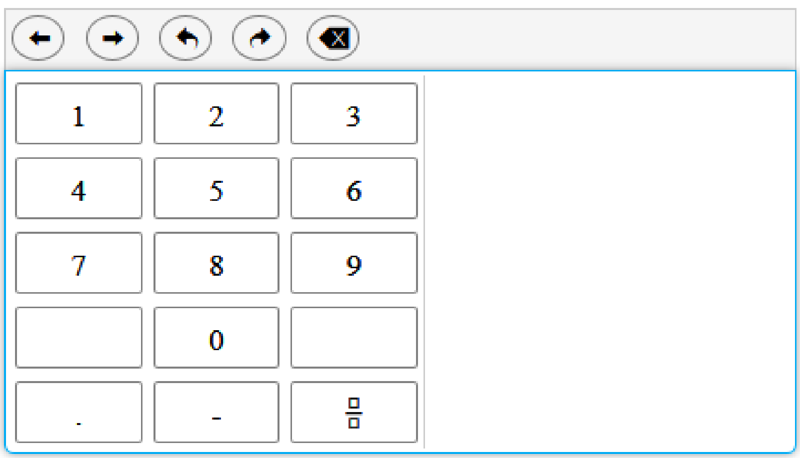
Sample Response: 1 point

Angle ABC is inscribed in a circle as shown.



What is the measure, in degrees, of $\angle ABC$?

$\frac{120}{2}$ *degrees*

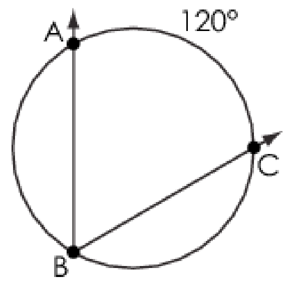


Notes on Scoring

This response earns full credit (1 point) because it shows a measure that is equivalent to the correct measure of the inscribed angle, 60º.

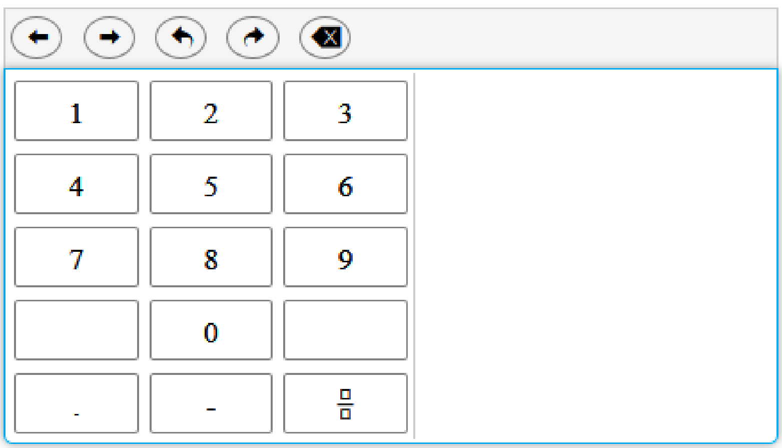
Sample Response: 0 points

Angle ABC is inscribed in a circle as shown.



What is the measure, in degrees, of $\angle ABC$?

240 *degrees*

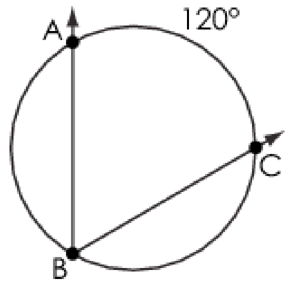


Notes on Scoring

This response earns no credit (0 points) because it shows an incorrect measure of the inscribed angle. The student may think that the measure of an angle inscribed in the circle is twice the measure of its intercepted arc and multiplies 120 by 2 to get 240°.

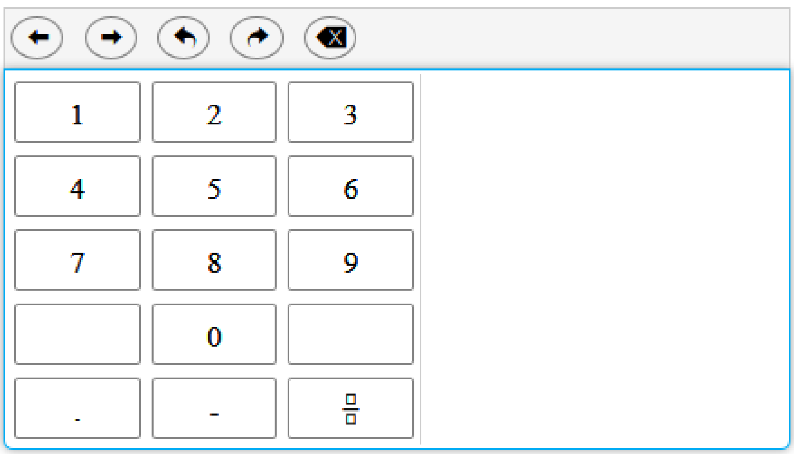
Sample Response: 0 points

Angle ABC is inscribed in a circle as shown.



What is the measure, in degrees, of $\angle ABC$?

120 *degrees*



Notes on Scoring

This response earns no credit (0 points) because it shows an incorrect measure of the inscribed angle. The student may confuse the relationship between the measure of a central angle of a circle and its intercepted arc with the relationship between the measure of the inscribed angle and its intercepted arc and thinks that the measure of the inscribed angle equals the measure of its intercepted arc, or 120°.

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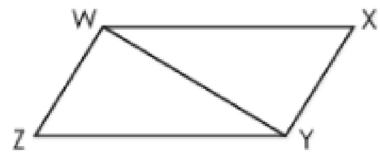
Spring 2018 Item Release

Question 21

Question and Scoring Guidelines

Question 21

A parallelogram and incomplete proof are shown.



Given: WXYZ is a parallelogram.
Prove: $\overline{WX} \cong \overline{YZ}$

Place reasons in the table to complete the proof.

Statements	Reasons
1. WXYZ is a parallelogram.	1. Given
2. $\overline{WX} \parallel \overline{YZ}$ $\overline{WZ} \parallel \overline{XY}$	2. Definition of a parallelogram
3. $\angle ZWY \cong \angle XYW$ $\angle ZYW \cong \angle XWY$	3.
4. $\overline{WY} \cong \overline{WY}$	4.
5. $\triangle WYZ \cong \triangle YWX$	5.
6. $\overline{WX} \cong \overline{YZ}$	6.

Corresponding angles are congruent.	SSS	Transitive property
Alternate exterior angles are congruent.	SAS	Reflexive property
Alternate interior angles are congruent.	ASA	Angle addition postulate
Corresponding parts of congruent triangles are congruent.	AA	Corresponding parts of congruent triangles are similar.

Points Possible: 1

Content Cluster: Prove geometric theorems.

Content Standard: Prove theorems about parallelograms. *Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals. (G.CO.11)*

Scoring Guidelines

Exemplar Response

Statements	Reasons
1. WXYZ is a parallelogram.	1. Given
2. $\overline{WX} \parallel \overline{YZ}$ $\overline{WZ} \parallel \overline{XY}$	2. Definition of a parallelogram
3. $\angle ZWY \cong \angle XYW$ $\angle ZYW \cong \angle XWY$	3. Alternate interior angles are congruent.
4. $\overline{WY} \cong \overline{WY}$	4. Reflexive property
5. $\triangle WYZ \cong \triangle YWX$	5. ASA
6. $\overline{WX} \cong \overline{YZ}$	6. Corresponding parts of congruent triangles are congruent.

Other Correct Responses

- N/A

For this item, a full-credit response includes:

- A correct proof (1 point).

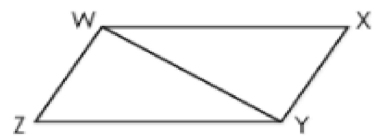
Geometry
Spring 2018 Item Release

Question 21

Sample Responses

Sample Response: 1 point

A parallelogram and incomplete proof are shown.



Given: WXYZ is a parallelogram.
Prove: $\overline{WX} \cong \overline{YZ}$

Place reasons in the table to complete the proof.

Statements	Reasons
1. WXYZ is a parallelogram.	1. Given
2. $\overline{WX} \parallel \overline{YZ}$ $\overline{WZ} \parallel \overline{XY}$	2. Definition of a parallelogram
3. $\angle ZWY \cong \angle XYW$ $\angle ZYW \cong \angle XWY$	3. Alternate interior angles are congruent.
4. $\overline{WY} \cong \overline{WY}$	4. Reflexive property
5. $\triangle WYZ \cong \triangle YWX$	5. ASA
6. $\overline{WX} \cong \overline{YZ}$	6. Corresponding parts of congruent triangles are congruent.

Corresponding angles are congruent.	SSS	Transitive property
Alternate exterior angles are congruent.	SAS	
		Angle addition postulate
	AA	Corresponding parts of congruent triangles are similar.

Notes on Scoring

This response earns full credit (1 point) because it shows the correct placement of four reasons to complete the proof.

Angles ZWY and XYW are congruent because they are alternate interior angles formed by the pair of parallel sides WZ and XY cut by the transversal WY. Likewise, angles ZYW and XWY are also congruent because they are alternate interior angles (Line 3) formed by the pair of parallel sides WX and ZY cut by the same transversal WY.

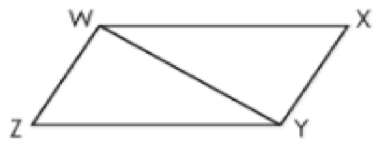
Side WY is congruent to itself by the Reflexive Property (Line 4).

Now, the proof shows enough evidence to claim that $\Delta WYZ \cong \Delta YWX$ by ASA, or the Angle-Side-Angle Congruence Theorem (Line 5).

Since the two triangles are congruent, their corresponding parts, such as sides WX and YZ, are also congruent. It is supported by the reason Corresponding Parts of Congruent Triangles are Congruent (Line 6).

Sample Response: 0 points

A parallelogram and incomplete proof are shown.



Given: WXYZ is a parallelogram.
Prove: $\overline{WX} \cong \overline{YZ}$

Place reasons in the table to complete the proof.

Statements	Reasons
1. WXYZ is a parallelogram.	1. Given
2. $\overline{WX} \parallel \overline{YZ}$ $\overline{WZ} \parallel \overline{XY}$	2. Definition of a parallelogram
3. $\angle ZWY \cong \angle XYW$ $\angle ZYW \cong \angle XWY$	3. Alternate interior angles are congruent.
4. $\overline{WY} \cong \overline{WY}$	4. Transitive property
5. $\triangle WYZ \cong \triangle YWX$	5. ASA
6. $\overline{WX} \cong \overline{YZ}$	6. Corresponding parts of congruent triangles are congruent.

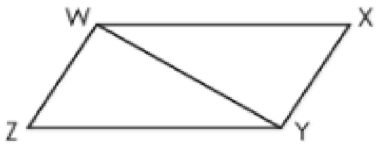
Corresponding angles are congruent.	SSS	
Alternate exterior angles are congruent.	SAS	Reflexive property
		Angle addition postulate
	AA	Corresponding parts of congruent triangles are similar.

Notes on Scoring

This response earns no credit (0 points) because it shows one incorrectly placed reason out of four reasons to complete the proof. The student may incorrectly select the Transitive Property instead of Reflexive Property to justify the congruence of line segment WY to itself.

Sample Response: 0 points

A parallelogram and incomplete proof are shown.



Given: WXYZ is a parallelogram.
Prove: $\overline{WX} \cong \overline{YZ}$

Place reasons in the table to complete the proof.

Statements	Reasons
1. WXYZ is a parallelogram.	1. Given
2. $\overline{WX} \parallel \overline{YZ}$ $\overline{WZ} \parallel \overline{XY}$	2. Definition of a parallelogram
3. $\angle ZWY \cong \angle XYW$ $\angle ZYW \cong \angle XWY$	3. Alternate interior angles are congruent.
4. $\overline{WY} \cong \overline{WY}$	4. Reflexive property
5. $\triangle WYZ \cong \triangle YWX$	5. ASA
6. $\overline{WX} \cong \overline{YZ}$	6. Corresponding parts of congruent triangles are similar.

Corresponding angles are congruent.	SSS	Transitive property
Alternate exterior angles are congruent.	SAS	
		Angle addition postulate
Corresponding parts of congruent triangles are congruent.	AA	

Notes on Scoring

This response earns no credit (0 points) because it shows one incorrectly placed reason out of four reasons to complete the proof. The student may incorrectly select Corresponding Parts of Congruent Triangles are Similar instead of Corresponding Parts of Congruent Triangles are Congruent to justify the congruence of a line segment WX and YZ.

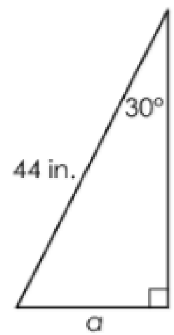
Geometry
Spring 2018 Item Release

Question 23

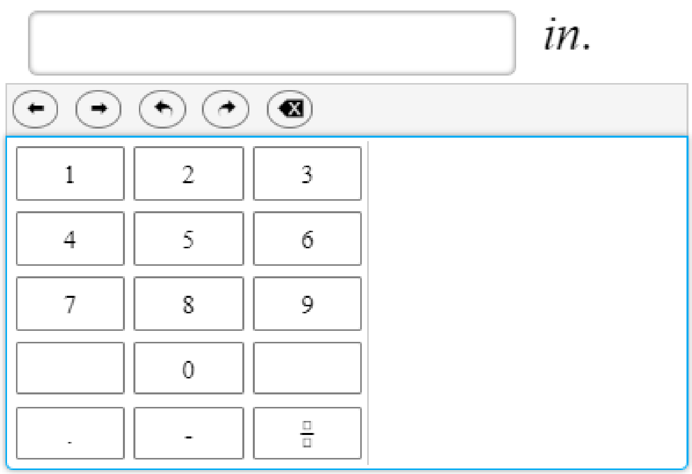
Question and Scoring Guidelines

Question 23

A triangle is shown.



What is the length, in inches (in.), of side *a*?



Points Possible: 1

Content Cluster: Define trigonometric ratios and solve problems involving right triangles.

Content Standard: Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems. *(G.SRT.8)*

Scoring Guidelines

Exemplar Response

- 22

Other Correct Responses

- Any equivalent value

For this item, a full-credit response includes:

- The correct length (1 point).

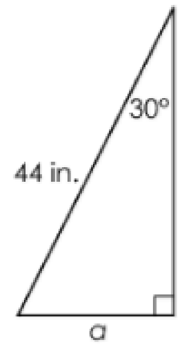
Geometry
Spring 2018 Item Release

Question 23

Sample Responses

Sample Response: 1 point

A triangle is shown.



What is the length, in inches (in.), of side *a*?

22 *in.*

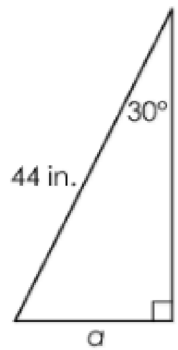
Notes on Scoring

This response earns full credit (1 point) because it shows the correct side length. Since the unknown side, *a*, is opposite to the given angle of 30º and the length of hypotenuse is 44 in., the right triangle definition of sine can be used to find the unknown side length.

In right triangles, the sine of an acute angle is the ratio of the length of the opposite leg to the length of hypotenuse. Based on this definition, sin (30º) = $\frac{a}{44}$. After multiplying both sides of this equation by 44, $a = 44\sin(30°)$, or $a = 22$ in.

Sample Response: 1 point

A triangle is shown.



What is the length, in inches (in.), of side *a*?

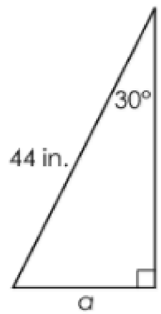
$\frac{44}{2}$ *in.*

Notes on Scoring

This response earns full credit (1 point) because it shows a value equivalent to 22.

Sample Response: 0 points

A triangle is shown.



What is the length, in inches (in.), of side *a*?

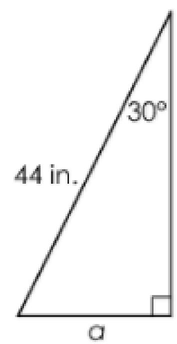
38.1 *in.*

Notes on Scoring

This response earns no credit (0 points) because it shows an incorrect value for the side length, *a*. The student may confuse the definition of sine and cosine to write an incorrect equation.

Sample Response: 0 points

A triangle is shown.



What is the length, in inches (in.), of side *a*?

$\frac{1}{88}$ *in.*

Notes on Scoring

This response earns no credit (0 points) because it shows an incorrect value for the side length, *a*. The student may have correctly set up the equation sin 30 = $\frac{a}{44}$, but then he or she may incorrectly divide both sides of the equation by 44 instead of multiplying by 44 when attempting to solve the equation for *a*.

Geometry
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Question 29

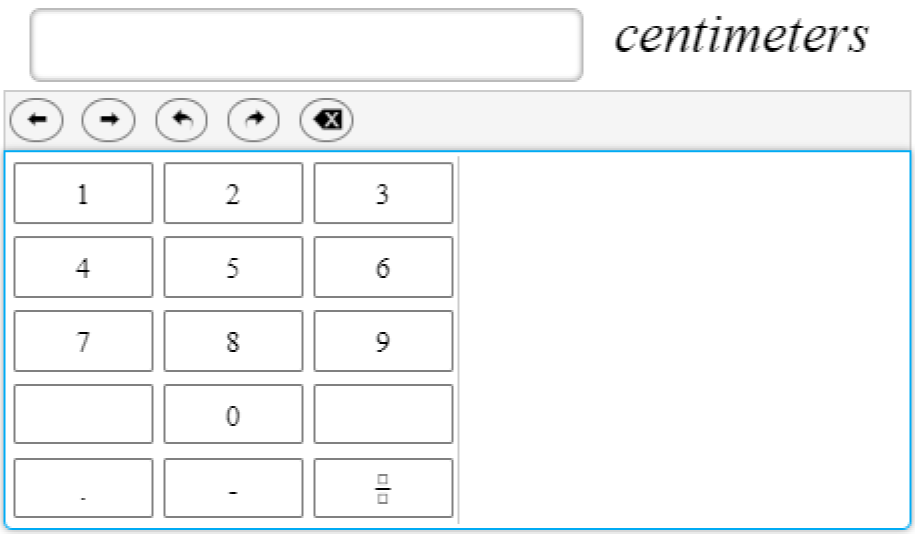
Question and Scoring Guidelines

Question 29

Points A, B and C lie on a circle with center Q.

- The area of sector AQB is twice the area of sector BQC.
- The length of arc AB is 28 centimeters.

What is the length, in centimeters, of arc BC?



Points Possible: 1

Content Cluster: Find arc lengths and areas of sectors of circles.

Content Standard: Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector. *(G.C.5)*

Scoring Guidelines

<u>Exemplar Response</u>

- 14

<u>Other Correct Responses</u>

- Any equivalent value

For this item, a full-credit response includes:

- The correct length (1 point).

Geometry
Spring 2018 Item Release

Question 29

Sample Responses

Sample Response: 1 point

Points A, B and C lie on a circle with center Q.

- The area of sector AQB is twice the area of sector BQC.
- The length of arc AB is 28 centimeters.

What is the length, in centimeters, of arc BC?

14 *centimeters*

Notes on Scoring

This response earns full credit (1 point) because it shows a correct length of arc BC.

In the same circle, if the area of the first circular sector is twice the area of the second sector, or in a 2:1 ratio, then the measures of central angles forming the sectors as well as the lengths of their corresponding arcs are also related as 2:1.

If the length of arc AB, which contains sector AQB, is 28 cm, then the length of arc BC, which contains sector BQC, is half of 28 cm, or 14 cm.

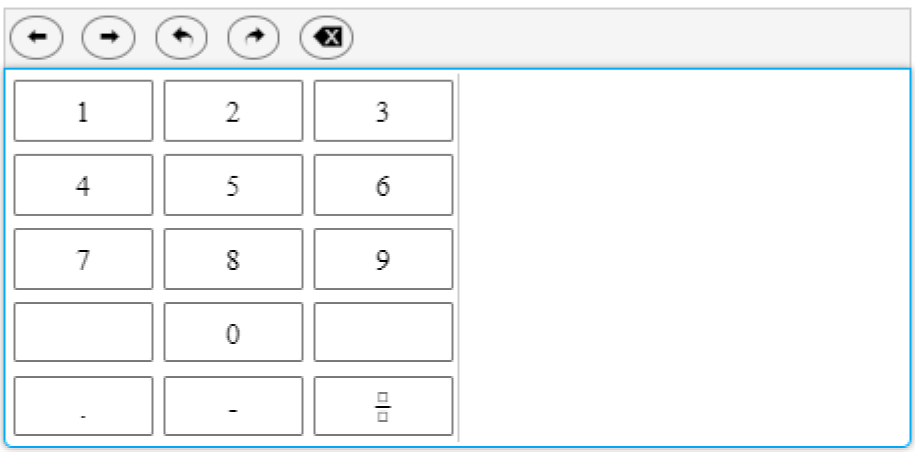
Sample Response: 1 point

Points A, B and C lie on a circle with center Q.

- The area of sector AQB is twice the area of sector BQC.
- The length of arc AB is 28 centimeters.

What is the length, in centimeters, of arc BC?

$\frac{28}{2}$ *centimeters*



Notes on Scoring

This response earns full credit (1 point) because it shows a correct length of arc BC equivalent to 14 cm.

Sample Response: 0 points

Points A, B and C lie on a circle with center Q.

- The area of sector AQB is twice the area of sector BQC.
- The length of arc AB is 28 centimeters.

What is the length, in centimeters, of arc BC?

56 *centimeters*

Notes on Scoring

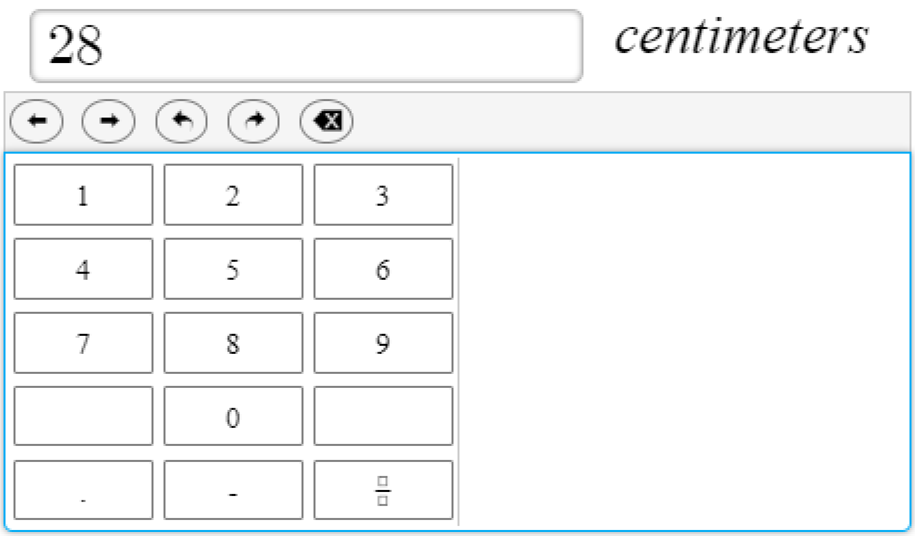
This response earns no credit (0 points) because it shows an incorrect length of arc BC. The student may think that if the length of arc AB, which contains sector AQB, is 28 cm, then the length of arc BC, which contains sector BQC, is twice the length of arc AB, or 28 • 2 = 56, instead of half of the length of arc AB, or 14 cm.

Sample Response: 0 points

Points A, B and C lie on a circle with center Q.

- The area of sector AQB is twice the area of sector BQC.
- The length of arc AB is 28 centimeters.

What is the length, in centimeters, of arc BC?



Notes on Scoring

This response earns no credit (0 points) because it shows an incorrect length of arc BC. The student may think that the length of arc BC would be equal to the length of arc AB, or 28 cm.

Geometry
Spring 2018 Item Release

Question 31

Question and Scoring Guidelines

Question 31

Which term is defined as two intersecting lines that form four right angles in a plane?

Ⓐ skew lines

Ⓑ straight lines

Ⓒ parallel lines

Ⓓ perpendicular lines

Points Possible: 1

Content Cluster: Experiment with transformations in the plane.

Content Standard: Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc. *(G.CO.1)*

Scoring Guidelines

Rationale for Option A: The student may confuse skew lines, which are in different planes and never intersect, with perpendicular lines, which are in the same plane and intersect forming four right angles.

Rationale for Option B: The student may confuse straight lines with perpendicular lines and focuses only on lines being straight instead of lines making 90-degree angles.

Rationale for Option C: The student may confuse parallel lines, which are in the same plane and never intersect, with perpendicular lines, which intersect and form four right angles.

Rationale for Option D: **Key** – The student correctly identifies that perpendicular lines are defined as two intersecting lines that form four right angles in a plane.

Sample Response: 1 point

Which term is defined as two intersecting lines that form four right angles in a plane?

Ⓐ skew lines

Ⓑ straight lines

Ⓒ parallel lines

● perpendicular lines

Geometry
Spring 2018 Item Release

Question 34

Question and Scoring Guidelines

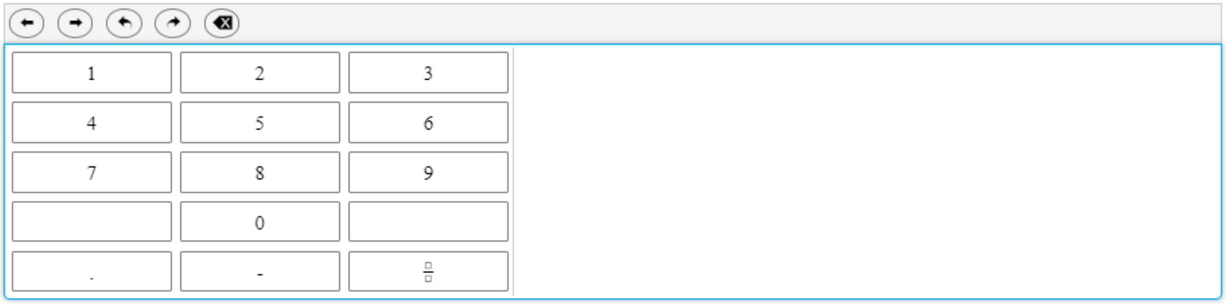
Question 34

A cone and a sphere have the same volume. The height of the cone is 96 units.

What could be the values for the radius of the cone and the sphere? Round your answers to the nearest hundredth as needed.

Radius of Cone: ____ *units*

Radius of Sphere: ____ *units*



Points Possible: 1

Content Cluster: Explain volume formulas and use them to solve problems.

Content Standard: Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems. *(G.GMD.3)*

Scoring Guidelines

Exemplar Response

- Radius of cone: 3 units
- Radius of sphere: 6 units

Other Correct Responses

- Any values for which $|24(\text{radius of cone})^2 - (\text{radius of sphere})^3| \leq 1$

For this item, a full-credit response includes:

- Two correct values (1 point).

Geometry
Spring 2018 Item Release

Question 34

Sample Responses

Sample Response: 1 point

A cone and a sphere have the same volume. The height of the cone is 96 units.

What could be the values for the radius of the cone and the sphere? Round your answers to the nearest hundredth as needed.

Radius of Cone: 3 *units*

Radius of Sphere: 6 *units*

Notes on Scoring

This response earns full credit (1 point) because it shows correct values for the radius of cone (R) and the radius of sphere (r). In this situation, the student can calculate these values by creating an equation that shows the equality of these two volumes as shown below.

Volume of cone = Volume of sphere
$\frac{1}{3}\pi R^2 h = \frac{4}{3}\pi r^3$

Using the fact that the height, h, of the cone is 96 units, the equation becomes $\frac{1}{3}\pi R^2 96 = \frac{4}{3}\pi r^3$.

After multiplying both sides by $\frac{3}{4}$ and then dividing both sides by π, the equation becomes $24R^2 = r^3$.

To find solutions to this equation, the student can pick any positive value for the radius of one shape, substitute it into the equation, and calculate the radius of another shape by solving the equation for the unknown. For example, if R = 3, then $24 \bullet 3^2 = r^3$, 216 = r^3, and $r = 6$; or if R = 5, then $24 \bullet 5^2 = r^3,$ and $r \approx 8.43$.

The acceptable values are those that satisfy the condition $|24R^2 - r^3| \leq 1$. Values for both radii can be rounded to the nearest hundredths of units, if necessary.

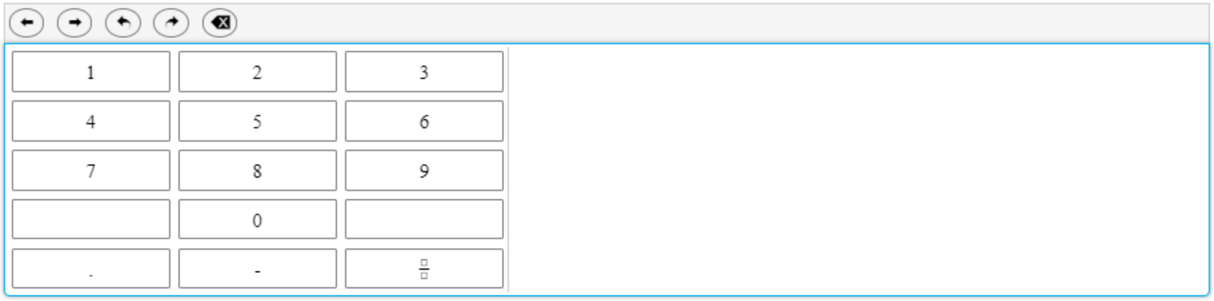
Sample Response: 1 point

A cone and a sphere have the same volume. The height of the cone is 96 units.

What could be the values for the radius of the cone and the sphere? Round your answers to the nearest hundredth as needed.

Radius of Cone: 3.0 *units*

Radius of Sphere: 6.0 *units*



Notes on Scoring

This response earns full credit (1 point) because it shows correct possible values for the radius of the cone (R) and the radius of the sphere (r).

Sample Response: 0 points

A cone and a sphere have the same volume. The height of the cone is 96 units.

What could be the values for the radius of the cone and the sphere? Round your answers to the nearest hundredth as needed.

Radius of Cone: 3 *units*

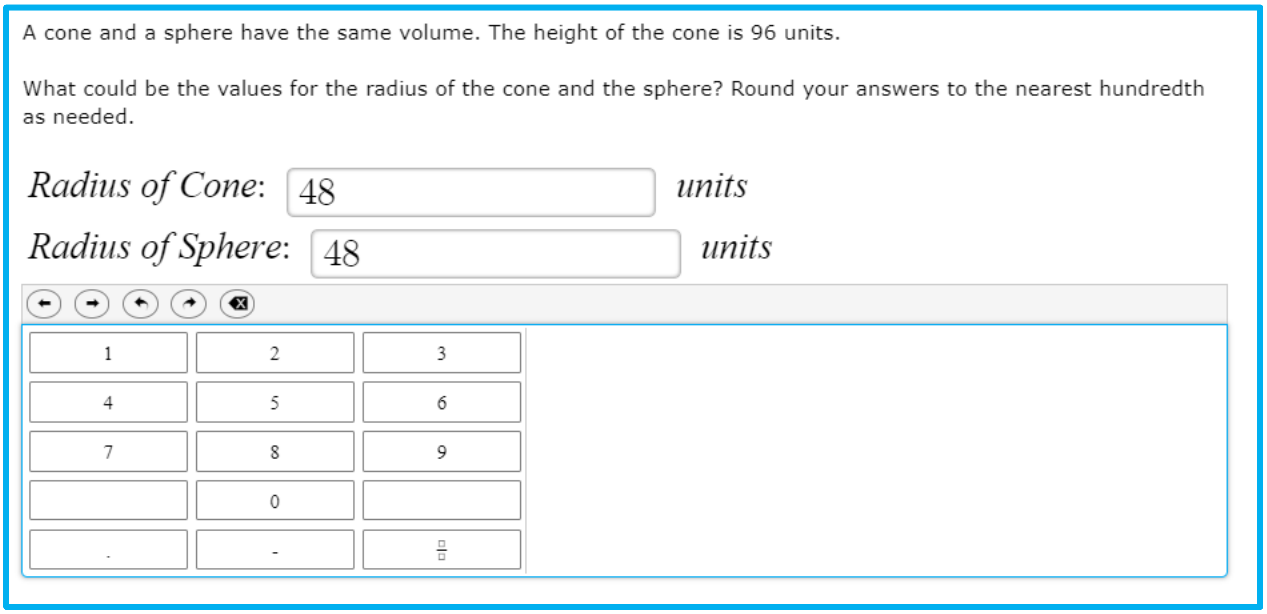
Radius of Sphere: 3 *units*

1	2	3
4	5	6
7	8	9
	0	
.	-	$\frac{\square}{\square}$

Notes on Scoring

This response earns no credit (0 points) because the volume of a cone with radius 3 units and height 96 units is not equal to the volume of the sphere with radius 3 units.

Sample Response: 0 points



Notes on Scoring

This response earns no credit (0 point) because the volume of a cone with radius 48 units and height 96 units is not equal to the volume of the sphere with radius 48 units. The student may confuse the height of the cone with the diameter of the cone and divides 96 by 2 to get the radius of 48. He or she may also incorrectly think that the radius of the cone and the radius of the sphere are equal and enters 48 for the radius of the sphere as well.

Geometry
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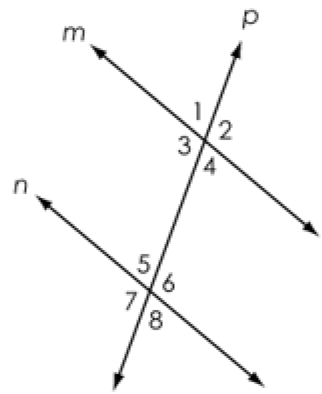
Question 39

Question and Scoring Guidelines

Question 39

Given: $m \parallel n$ and transversal p

Prove: $\angle 5 \cong \angle 4$



Part of a proof is shown. Place statements and reasons in the table to complete the proof.

Statements	Reasons
1. $m \parallel n$ and transversal p	1. Given
2.	2.
3.	3.
4. $\angle 5 \cong \angle 4$	4.

$\angle 8 \cong \angle 1$	Vertical angles theorem
$\angle 1 \cong \angle 4$	Corresponding angles postulate
$\angle 8 \cong \angle 4$	Transitive property
$\angle 5 \cong \angle 8$	Alternate exterior angles theorem
$\angle 5 \cong \angle 7$	Reflexive property
$\angle 4 \cong \angle 7$	Angle addition postulate

Points Possible: 1

Content Cluster: Prove geometric theorems.

Content Standard: Prove theorems about lines and angles. *Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints. (G.CO.9)*

Scoring Guidelines

Exemplar Response

Statements	Reasons
1. $m \parallel n$ and transversal p	1. Given
2. $\angle 8 \cong \angle 4$	2. Corresponding angles postulate
3. $\angle 5 \cong \angle 8$	3. Vertical angles theorem
4. $\angle 5 \cong \angle 4$	4. Transitive property

Other Correct Responses

Statements	Reasons
1. $m \parallel n$ and transversal p	1. Given
2. $\angle 5 \cong \angle 8$ $\angle 1 \cong \angle 4$	2. Vertical angles theorem
3. $\angle 8 \cong \angle 1$	3. Alternate exterior angles theorem
4. $\angle 5 \cong \angle 4$	4. Transitive property

- Lines 2 and 3 can be interchanged in the Exemplar and Other Correct Responses

For this item, a full-credit response includes:

- The correct proof (1 point).

Geometry
Spring 2018 Item Release

Question 39

Sample Responses

Sample Response: 1 point

Part of a proof is shown. Place statements and reasons in the table to complete the proof.

Statements	Reasons
1. $m \parallel n$ and transversal p	1. Given
2. $\angle 1 \cong \angle 4$ $\angle 5 \cong \angle 8$	2. Vertical angles theorem
3. $\angle 8 \cong \angle 1$	3. Alternate exterior angles theorem
4. $\angle 5 \cong \angle 4$	4. Transitive property

Reflexive property

Corresponding angles postulate

$\angle 8 \cong \angle 4$

$\angle 5 \cong \angle 7$

$\angle 4 \cong \angle 7$ Angle addition postulate

Notes on Scoring

This response earns full credit (1 point) because it shows a placement of five correct answer options—two statements and three reasons—to complete the proof.

As shown in the diagram, lines *m*, *n* and *p* form four pairs of vertical angles. By the Vertical Angles Theorem, vertical angles are congruent. Therefore, statement #2 is $\angle 1 \cong \angle 4$ and $\angle 5 \cong \angle 8$ and reason #2 is Vertical Angles Theorem.

Since lines *m* and *n* are parallel and *p* is a transversal (given), the pairs of alternate exterior angles are congruent. Therefore, statement #3 is $\angle 8 \cong \angle 1$, and reason #3 is Alternate Exterior Angles Theorem.

The use of the Transitive Property (reason #4) completes the proof because statement #4 shows $\angle 5 \cong \angle 4$ which follows from two previous statements.

Sample Response: 1 point

Part of a proof is shown. Place statements and reasons in the table to complete the proof.

Statements	Reasons
1. $m \parallel n$ and transversal p	1. Given
2. $\angle 5 \cong \angle 8$	2. Vertical angles theorem
3. $\angle 8 \cong \angle 4$	3. Corresponding angles postulate
4. $\angle 5 \cong \angle 4$	4. Transitive property

$\angle 8 \cong \angle 1$	
$\angle 1 \cong \angle 4$	
	Alternate exterior angles theorem
$\angle 5 \cong \angle 7$	Reflexive property
$\angle 4 \cong \angle 7$	Angle addition postulate

Notes on Scoring

This response earns full credit (1 point) because it shows a placement of five correct answer options—two statements and three reasons—to complete the proof.

Since $\angle 5 \cong \angle 8$ (statement #2) by the Vertical Angles Theorem and $\angle 8 \cong \angle 4$ (statement #3) by the Corresponding Angles Postulate, then $\angle 5 \cong \angle 4$ (statement #4) by the Transitive Property (reason #4).

Sample Response: 0 points

Part of a proof is shown. Place statements and reasons in the table to complete the proof.

Statements	Reasons
1. $m \parallel n$ and transversal p	1. Given
2. $\angle 5 \cong \angle 8$	2. Vertical angles theorem
3. $\angle 8 \cong \angle 4$	3. Alternate exterior angles theorem
4. $\angle 5 \cong \angle 4$	4. Transitive property

$\angle 8 \cong \angle 1$	
$\angle 1 \cong \angle 4$	Corresponding angles postulate
$\angle 5 \cong \angle 7$	Reflexive property
$\angle 4 \cong \angle 7$	Angle addition postulate

Notes on Scoring

This response earns no credit (0 points) because it shows the placement of one incorrect reason (reason #3) in an attempt to complete the proof. The student may confuse the Corresponding Angles Postulate with the Alternate Exterior Angles Postulate.

Sample Response: 0 points

Part of a proof is shown. Place statements and reasons in the table to complete the proof.

Statements	Reasons
1. $m \parallel n$ and transversal p	1. Given
2. $\angle 1 \cong \angle 4$ $\angle 5 \cong \angle 8$	2. Vertical angles theorem
3. $\angle 8 \cong \angle 1$	3. Alternate exterior angles theorem
4. $\angle 5 \cong \angle 4$	4. Reflexive property

	Corresponding angles postulate
$\angle 8 \cong \angle 4$	Transitive property
$\angle 5 \cong \angle 7$	
$\angle 4 \cong \angle 7$	Angle addition postulate

Notes on Scoring

This response earns no credit (0 points) because it shows a placement of one incorrect reason (reason #4) in an attempt to complete the proof. The student may confuse the Reflexive Property with the Transitive Property.

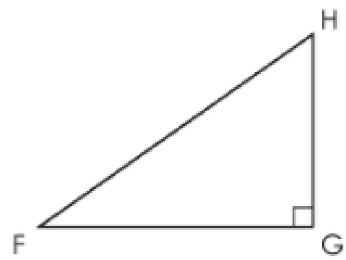
Geometry
Spring 2018 Item Release

Question 40

Question and Scoring Guidelines

Question 40

Right triangle FHG is shown.



The sine of ∠F is 0.53.

What is the cosine of ∠H? Round your answer to the nearest hundredth as needed.

Points Possible: 1

Content Cluster: Define trigonometric ratios and solve problems involving right triangles.

Content Standard: Explain and use the relationship between the sine and cosine of complementary angles. *(G.SRT.7)*

Scoring Guidelines

Exemplar Response

- 0.53

Other Correct Responses

- Any equivalent value

For this item, a full-credit response includes:

- A correct value (1 point).

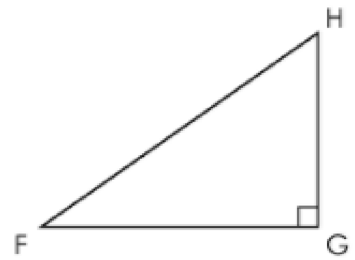
Geometry
Spring 2018 Item Release

Question 40

Sample Responses

Sample Response: 1 point

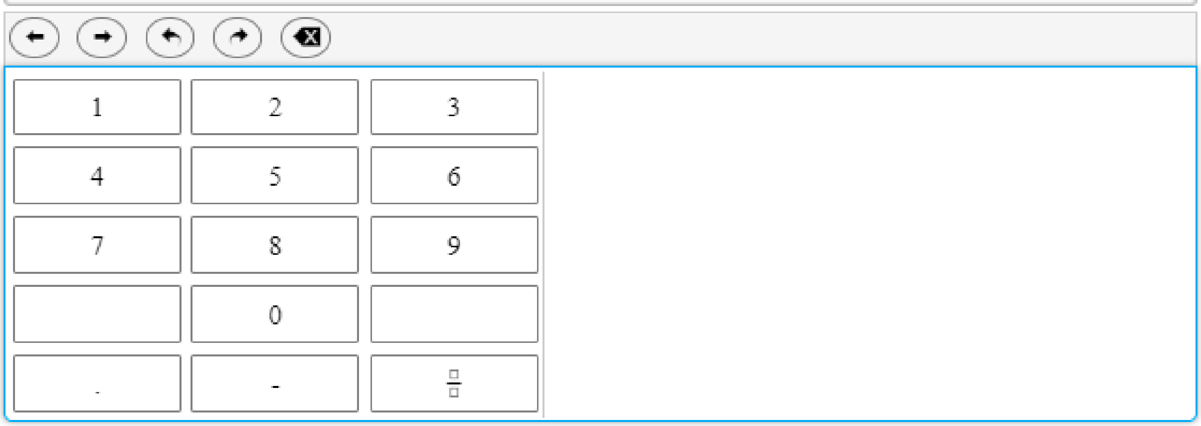
Right triangle FHG is shown.



The sine of ∠F is 0.53.

What is the cosine of ∠H? Round your answer to the nearest hundredth as needed.

0.53

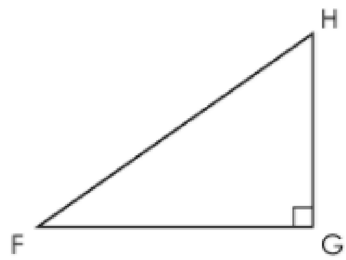


Notes on Scoring

This response earns full credit (1 point) because it shows the correct value for cos(H). In right triangles, the sine of an acute angle equals the cosine of the angle complementary to angle F. Since angles F and H are complementary angles (their sum is 90º), the cosine of H equals the sine of F, or 0.53.

Sample Response: 1 point

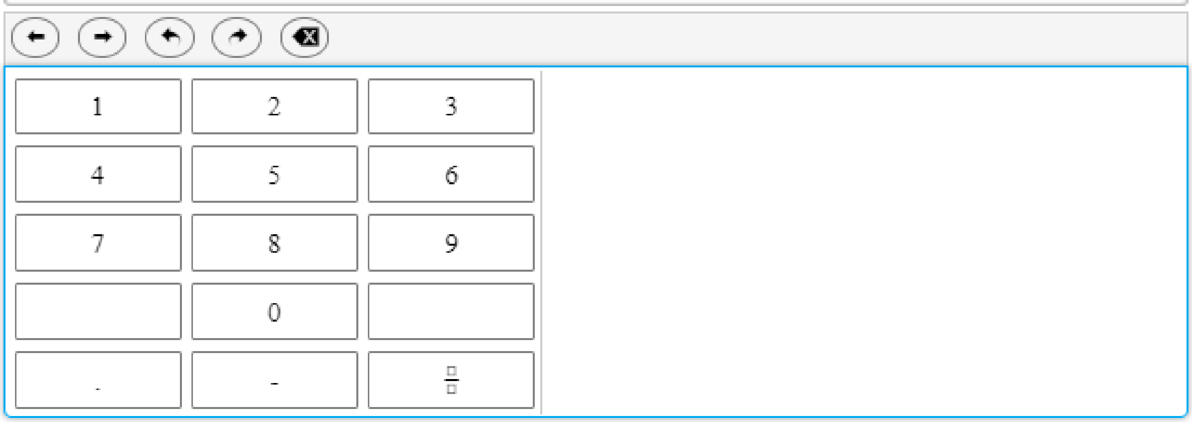
Right triangle FHG is shown.



The sine of ∠F is 0.53.

What is the cosine of ∠H? Round your answer to the nearest hundredth as needed.

$$\frac{53}{100}$$

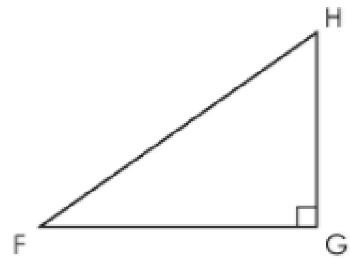


Notes on Scoring

This response earns full credit (1 point) because it shows a correct equivalent value for cos(H).

Sample Response: 0 points

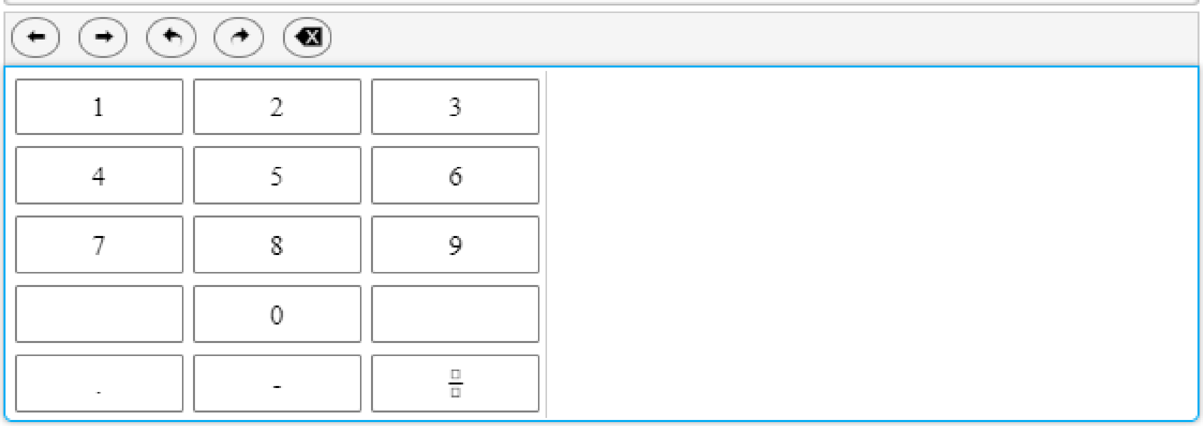
Right triangle FHG is shown.



The sine of ∠F is 0.53.

What is the cosine of ∠H? Round your answer to the nearest hundredth as needed.

$$\frac{1}{0.53}$$

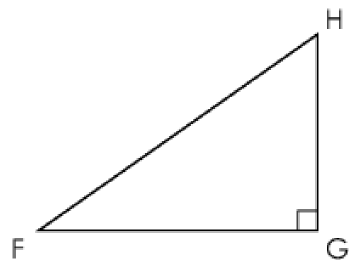


Notes on Scoring

This response earns no credit (0 points) because it shows an incorrect value for cos(H). The student may incorrectly think that the cos(H) is the reciprocal of sin(F) because the angles are complementary.

Sample Response: 0 points

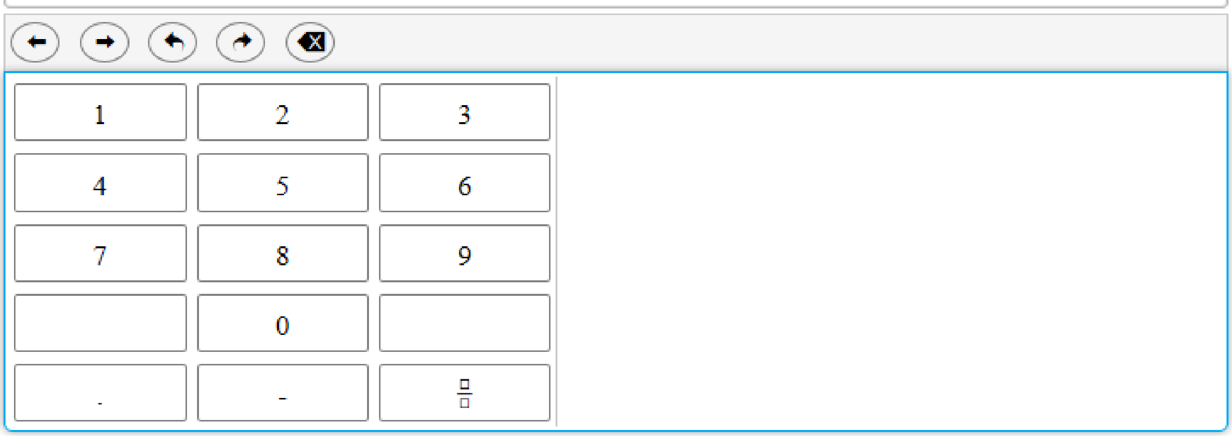
Right triangle FHG is shown.



The sine of ∠F is 0.53.

What is the cosine of ∠H? Round your answer to the nearest hundredth as needed.

0.47



Notes on Scoring

This response earns no credit (0 points) because it shows the incorrect value for cos(H). The student may incorrectly think that the cos(H) = 1 – sin(F) because the angles are complementary.

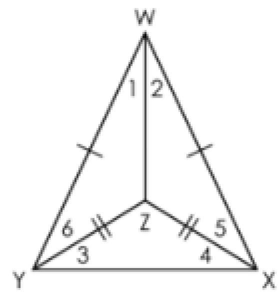
Geometry
Spring 2018 Item Release

Question 44

Question and Scoring Guidelines

Question 44

Triangle YWX is shown.



Given: $\overline{WY}\cong\overline{WX}$, $\overline{ZY}\cong\overline{ZX}$

Prove: $\overline{WZ}$ bisects $\angle YWX$

Place statements and reasons in the blank boxes to complete the proof.

Statements	Reasons
$\overline{WY}\cong\overline{WX}$ $\overline{ZY}\cong\overline{ZX}$	Given
$\angle WYX\cong\angle WXY$ $\angle 3\cong\angle 4$	
$m\angle WYX=m\angle WXY$ $m\angle 3=m\angle 4$	Measures of congruent angles are equal.
$m\angle WYX=m\angle 6+m\angle 3$ $m\angle WXY=m\angle 5+m\angle 4$	
$m\angle 6+m\angle 3=m\angle 5+m\angle 4$	Substitution
	Substitution
$m\angle 6=m\angle 5$	
	SAS
$\angle YWZ\cong\angle XWZ$	
$\overline{WZ}$ bisects $\angle YWX$	

$m\angle 6+m\angle 3=m\angle 5+m\angle 3$	$\triangle WYZ\cong\triangle WXZ$	Addition Property of Equality
$m\angle 6=m\angle 5+m\angle 4-m\angle 3$	$\triangle WYX\cong\triangle ZYX$	Substitution
$m\angle 6+m\angle 3=m\angle 3+m\angle 4$	Corresponding parts of congruent triangles are congruent.	Angle Addition Postulate
Base angles of isosceles triangles are congruent.	Definition of angle bisector	Reflexive Property
Corresponding parts of similar triangles are congruent.		

Points Possible: 2

Content Cluster: Prove geometric theorems.

Content Standard: Prove theorems about triangles. *Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point. (G.CO.10)*

Scoring Guidelines

Exemplar Response

Statements	Reasons
$\overline{WY} \cong \overline{WX}$ $\overline{ZY} \cong \overline{ZX}$	Given
$\angle WYX \cong \angle WXY$ $\angle 3 \cong \angle 4$	Base angles of isosceles triangles are congruent.
$m\angle WYX = m\angle WXY$	Measures of congruent angles are equal.
$m\angle WYX = m\angle 6 + m\angle 3$ $m\angle WXY = m\angle 5 + m\angle 4$	Angle Addition Postulate
$m\angle 6 + m\angle 3 = m\angle 5 + m\angle 4$	Substitution
$m\angle 6 + m\angle 3 = m\angle 5 + m\angle 3$	Substitution
$m\angle 6 = m\angle 5$	Addition Property of Equality
$\triangle WYZ \cong \triangle WXZ$	SAS
$\angle YWZ \cong \angle XWZ$	Corresponding parts of congruent triangles are congruent
$\overline{WZ}$ bisects $\angle YWX$	Definition of angle bisector

Other Correct Responses

- N/A

For this item, a full-credit response includes:

- four to six correctly filled in spaces (1 point)
  OR
- seven correctly filled in spaces (2 points).

Geometry
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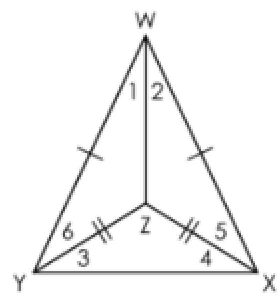
Question 44

Sample Responses

Sample Response: 2 points

Triangle YWX is shown.



Given: $\overline{WY} \cong \overline{WX}$, $\overline{ZY} \cong \overline{ZX}$

Prove: $\overline{WZ}$ bisects $\angle YWX$

Place statements and reasons in the blank boxes to complete the proof.

Statements	Reasons
$\overline{WY} \cong \overline{WX}$ $\overline{ZY} \cong \overline{ZX}$	Given
$\angle WYX \cong \angle WXY$ $\angle 3 \cong \angle 4$	Base angles of isosceles triangles are congruent.
$m\angle WYX = m\angle WXY$ $m\angle 3 = m\angle 4$	Measures of congruent angles are equal.
$m\angle WYX = m\angle 6 + m\angle 3$ $m\angle WXY = m\angle 5 + m\angle 4$	Angle Addition Postulate
$m\angle 6 + m\angle 3 = m\angle 5 + m\angle 4$	Substitution
$m\angle 6 + m\angle 3 = m\angle 5 + m\angle 3$	Substitution
$m\angle 6 = m\angle 5$	Addition Property of Equality
$\triangle WYZ \cong \triangle WXZ$	SAS
$\angle YWZ \cong \angle XWZ$	Corresponding parts of congruent triangles are congruent.
$\overline{WZ}$ bisects $\angle YWX$	Definition of angle bisector

$m\angle 6 = m\angle 5 + m\angle 4 - m\angle 3$	$\triangle WYX \cong \triangle ZYX$	Substitution
$m\angle 6 + m\angle 3 = m\angle 3 + m\angle 4$		
		Reflexive Property
Corresponding parts of similar triangles are congruent.		

Notes on Scoring

This response earns full credit (2 points) because it shows a placement of seven correct answer options—two statements and five reasons—to complete the proof.

Following from the given congruence of sides, ΔWYX and ΔZYX are isosceles. Therefore, $\angle WYX$ and $\angle WXY$ as well as $\angle 3$ and $\angle 4$ are congruent since base angles of isosceles triangles are congruent (reason for row 2).

The statement in row 4 shows that by the Angle Addition Postulate (reason for row 4) the measure of each base angle of ΔWYX is being restated as a sum of the measures of the two adjacent angles.

The missing Statement in row 6 must logically follow from the statement in row 5 and preconditions the next statement in row 7. By Substitution, the missing statement is $m\angle 6 + m\angle 3 = m\angle 5 + m\angle 3$.

Next, after using the Addition Property of Equality (reason for row 7) and adding $(-m\angle 3)$ to both sides, the equation becomes $m\angle 6 = m\angle 5$.

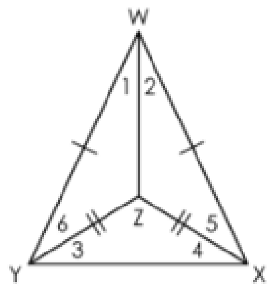
Now the proof shows enough evidence to claim $\Delta WYZ \cong \Delta WXZ$ (statement for row 8) by SAS, or the Side-Angle-Side Congruence Theorem.

Since triangles are congruent, angles $\angle YWZ$ and $\angle XWZ$ are also congruent since Corresponding Parts of Congruent Triangles are Congruent (reason for row 9).

Since two adjacent angles $\angle YWZ$ and $\angle XWZ$ are congruent, their common side WZ is the bisector of angle $\angle YWX$ by definition of an angle bisector (reason for row 10).

Sample Response: 1 point

Triangle YWX is shown.



Given: $\overline{WY} \cong \overline{WX}$, $\overline{ZY} \cong \overline{ZX}$

Prove: $\overline{WZ}$ bisects $\angle YWX$

Place statements and reasons in the blank boxes to complete the proof.

Statements	Reasons
$\overline{WY} \cong \overline{WX}$ $\overline{ZY} \cong \overline{ZX}$	Given
$\angle WYX \cong \angle WXY$ $\angle 3 \cong \angle 4$	Substitution
$m\angle WYX = m\angle WXY$ $m\angle 3 = m\angle 4$	Measures of congruent angles are equal.
$m\angle WYX = m\angle 6 + m\angle 3$ $m\angle WXY = m\angle 5 + m\angle 4$	Angle Addition Postulate
$m\angle 6 + m\angle 3 = m\angle 5 + m\angle 4$	Substitution
$m\angle 6 = m\angle 5 + m\angle 4 - m\angle 3$	Substitution
$m\angle 6 = m\angle 5$	Addition Property of Equality
$\triangle WYZ \cong \triangle WXZ$	SAS
$\angle YWZ \cong \angle XWZ$	Corresponding parts of similar triangles are congruent.
$\overline{WZ}$ bisects $\angle YWX$	Definition of angle bisector

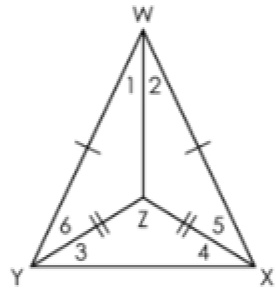
$m\angle 6 + m\angle 3 = m\angle 5 + m\angle 3$		
	$\triangle WYX \cong \triangle ZYX$	
$m\angle 6 + m\angle 3 = m\angle 3 + m\angle 4$	Corresponding parts of congruent triangles are congruent.	
Base angles of isosceles triangles are congruent.		Reflexive Property

Notes on Scoring

This response earns partial credit (1 point) because it shows a correct placement of four out of seven correct answer options. The sample response has an incorrect reason for row 2, an incorrect reason for row 10, and an incorrect statement for row 6.

Sample Response: 1 point

Triangle YWX is shown.



Given: $\overline{WY} \cong \overline{WX}$, $\overline{ZY} \cong \overline{ZX}$

Prove: $\overline{WZ}$ bisects $\angle YWX$

Place statements and reasons in the blank boxes to complete the proof.

Statements	Reasons
$\overline{WY} \cong \overline{WX}$ $\overline{ZY} \cong \overline{ZX}$	Given
$\angle WYX \cong \angle WXY$ $\angle 3 \cong \angle 4$	Base angles of isosceles triangles are congruent.
$m\angle WYX = m\angle WXY$ $m\angle 3 = m\angle 4$	Measures of congruent angles are equal.
$m\angle WYX = m\angle 6 + m\angle 3$ $m\angle WXY = m\angle 5 + m\angle 4$	Angle Addition Postulate
$m\angle 6 + m\angle 3 = m\angle 5 + m\angle 4$	Substitution
$m\angle 6 = m\angle 5 + m\angle 4 - m\angle 3$	Substitution
$m\angle 6 = m\angle 5$	Addition Property of Equality
$\triangle WYZ \cong \triangle WXZ$	SAS
$\angle YWZ \cong \angle XWZ$	Corresponding parts of similar triangles are congruent.
$\overline{WZ}$ bisects $\angle YWX$	Definition of angle bisector

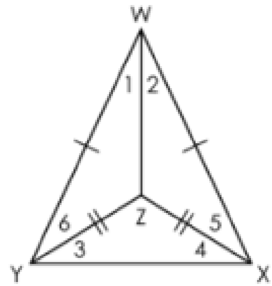
$m\angle 6 + m\angle 3 = m\angle 5 + m\angle 3$		
	$\triangle WYX \cong \triangle ZYX$	Substitution
$m\angle 6 + m\angle 3 = m\angle 3 + m\angle 4$	Corresponding parts of congruent triangles are congruent.	
		Reflexive Property

Notes on Scoring

This response earns partial credit (1 point) because it shows a correct placement of five out of seven correct answer options. The sample response has an incorrect statement for row 6 and an incorrect reason for row 10.

Sample Response: 0 points

Triangle YWX is shown.



Given: $\overline{WY} \cong \overline{WX}$, $\overline{ZY} \cong \overline{ZX}$

Prove: $\overline{WZ}$ bisects $\angle YWX$

Place statements and reasons in the blank boxes to complete the proof.

Statements	Reasons
$\overline{WY} \cong \overline{WX}$ $\overline{ZY} \cong \overline{ZX}$	Given
$\angle WYX \cong \angle WXY$ $\angle 3 \cong \angle 4$	Substitution
$m\angle WYX = m\angle WXY$ $m\angle 3 = m\angle 4$	Measures of congruent angles are equal.
$m\angle WYX = m\angle 6 + m\angle 3$ $m\angle WXY = m\angle 5 + m\angle 4$	Angle Addition Postulate
$m\angle 6 + m\angle 3 = m\angle 5 + m\angle 4$	Substitution
$m\angle 6 = m\angle 5 + m\angle 4 - m\angle 3$	Substitution
$m\angle 6 = m\angle 5$	Addition Property of Equality
$\triangle WYX \cong \triangle ZYX$	SAS
$\angle YWZ \cong \angle XWZ$	Corresponding parts of similar triangles are congruent.
$\overline{WZ}$ bisects $\angle YWX$	Definition of angle bisector

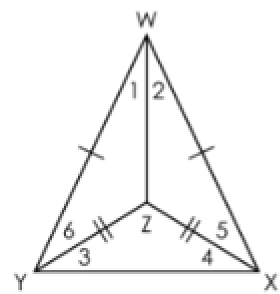
$m\angle 6 + m\angle 3 = m\angle 5 + m\angle 3$	$\triangle WYZ \cong \triangle WXZ$	
$m\angle 6 + m\angle 3 = m\angle 3 + m\angle 4$	Corresponding parts of congruent triangles are congruent.	
Base angles of isosceles triangles are congruent.		Reflexive Property

Notes on Scoring

This response earns no credit (0 points) because it shows a correct placement of three out of seven correct answer options. The sample response has an incorrect reason for rows 2 and 9, and an incorrect statement for rows 6 and 8.

Sample Response: 0 points

Triangle YWX is shown.



Given: $\overline{WY}\cong\overline{WX}$, $\overline{ZY}\cong\overline{ZX}$

Prove: $\overline{WZ}$ bisects $\angle YWX$

Place statements and reasons in the blank boxes to complete the proof.

Statements	Reasons
$\overline{WY}\cong\overline{WX}$ $\overline{ZY}\cong\overline{ZX}$	Given
$\angle WYX\cong\angle WXY$ $\angle 3\cong\angle 4$	Substitution
$m\angle WYX=m\angle WXY$ $m\angle 3=m\angle 4$	Measures of congruent angles are equal.
$m\angle WYX=m\angle 6+m\angle 3$ $m\angle WXY=m\angle 5+m\angle 4$	Addition Property of Equality
$m\angle 6+m\angle 3=m\angle 5+m\angle 4$	Substitution
$m\angle 6=m\angle 5+m\angle 4-m\angle 3$	Substitution
$m\angle 6=m\angle 5$	Angle Addition Postulate
$\triangle WYX\cong\triangle ZYX$	SAS
$\angle YWZ\cong\angle XWZ$	Corresponding parts of similar triangles are congruent.
$\overline{WZ}$ bisects $\angle YWX$	Definition of angle bisector

$m\angle 6+m\angle 3=m\angle 5+m\angle 3$	$\triangle WYZ\cong\triangle WXZ$	
$m\angle 6+m\angle 3=m\angle 3+m\angle 4$	Corresponding parts of congruent triangles are congruent.	
Base angles of isosceles triangles are congruent.		Reflexive Property

Notes on Scoring

This response earns no credit (0 points) because it shows a correct placement of only one out of seven correct answer options. The only correct placement is the reason for row 10.

Geometry
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Question 45

Question and Scoring Guidelines

Question 45

The equation of a line is shown.

$6x - 3y = 5$

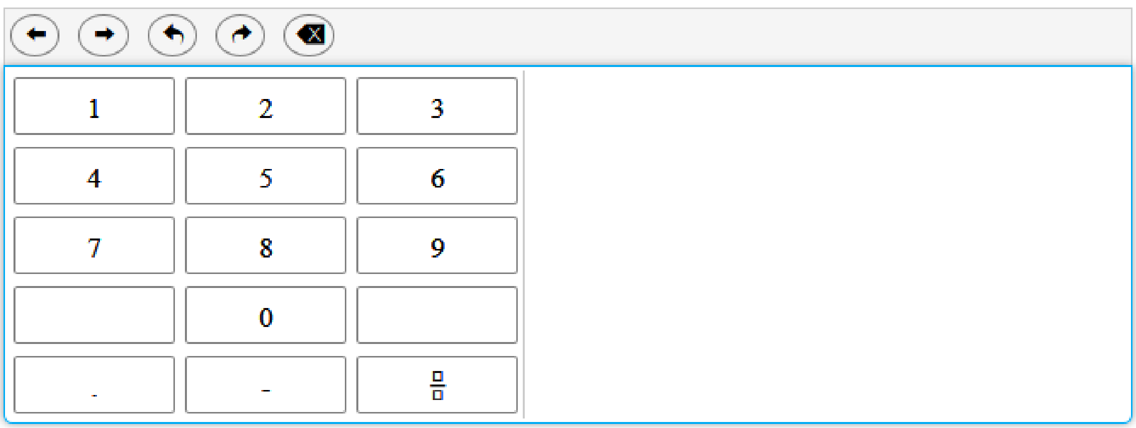
A dilation centered at the origin with a scale factor of 6 is applied to this line.

A. What is the slope of the line after the dilation?

B. What is the value of the y-intercept of the line after the dilation?

A.

B.



Points Possible: 2

Content Cluster: Understand similarity in terms of similarity transformations.

Content Standard: Verify experimentally the properties of dilations given by a center and a scale factor: (*G.SRT.1)*
a. A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.

Scoring Guidelines

Exemplar Response

- A. 2
  B. -10

Other Correct Responses

- Any equivalent values

For this item, a full-credit response includes:

- The correct slope (1 point)
  AND
- The correct *y*-intercept (1 point).

Geometry
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Question 45

Sample Responses

Sample Response: 2 points

The equation of a line is shown.

$6x - 3y = 5$

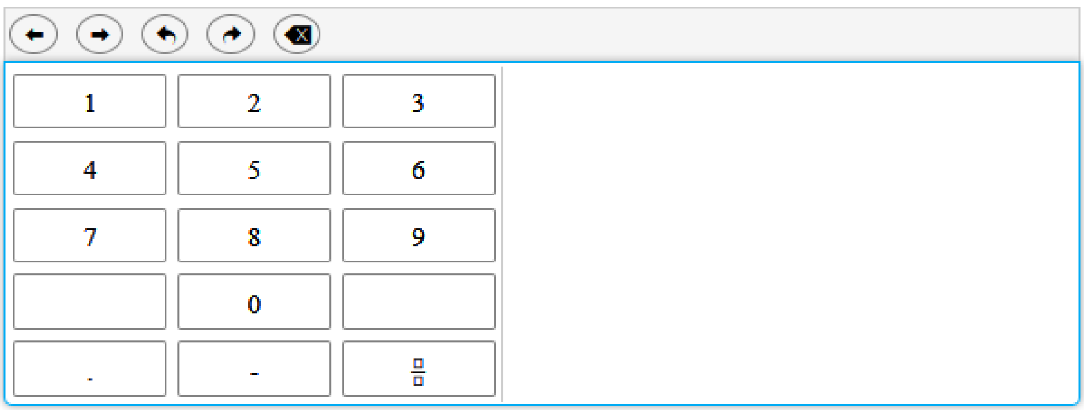
A dilation centered at the origin with a scale factor of 6 is applied to this line.

A. What is the slope of the line after the dilation?

B. What is the value of the y-intercept of the line after the dilation?

$A.$ 2.0

$B.$ -10.0



Notes on Scoring

This response earns full credit (2 points) because it shows a decimal form of the correct slope and y-intercept of the line after the dilation.

Sample Response: 2 points

The equation of a line is shown.

$6x - 3y = 5$

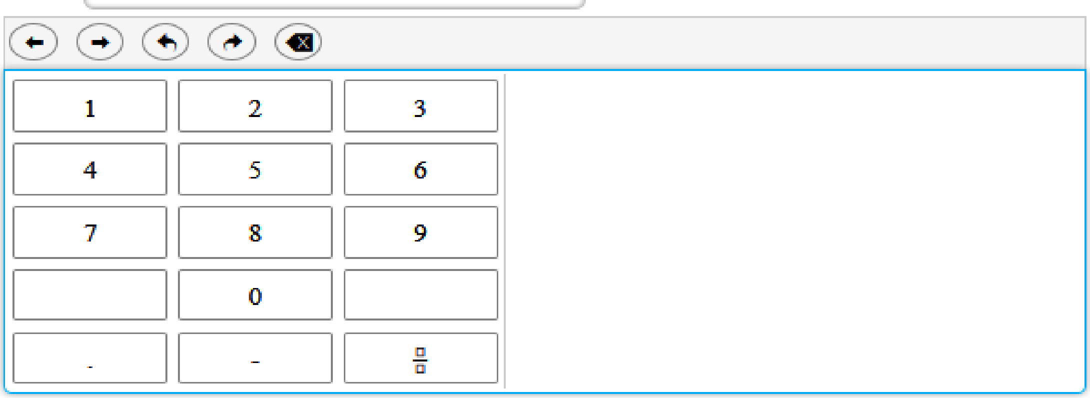
A dilation centered at the origin with a scale factor of 6 is applied to this line.

A. What is the slope of the line after the dilation?

B. What is the value of the y-intercept of the line after the dilation?

$A.$ $\frac{6}{3}$

$B.$ $-\frac{30}{3}$



Notes on Scoring

This response earns full credit (2 points) because it shows a correct slope and a correct y-intercept of the line after the dilation.

When the given equation of the line, $6x - 3y = 5$, is written in slope-intercept form, it becomes $y = 2x - \frac{5}{3}$. From here, the slope of the line is 2 and the y-intercept is located at the point $\left(0, -\frac{5}{3}\right)$.

If a dilation is applied to a line not passing through the center of dilation, it takes the line to a parallel line. Since parallel lines have equal slopes, the slope of the line after the dilation equals the slope of the original line, or 2. The correct response for part A is 2.

When a line is dilated with the center of dilation at the origin by a scale factor 6, the image of the y-intercept at $\left(0, -\frac{5}{3}\right)$ remains on the same side of the y-axis but 6 times as far from the origin or at $\left(-\frac{5}{3}\right) \bullet 6 = -10$. The value of the y-intercept of the dilated line, and the answer to part B, is –10.

Sample Response: 1 point

The equation of a line is shown.

$6x - 3y = 5$

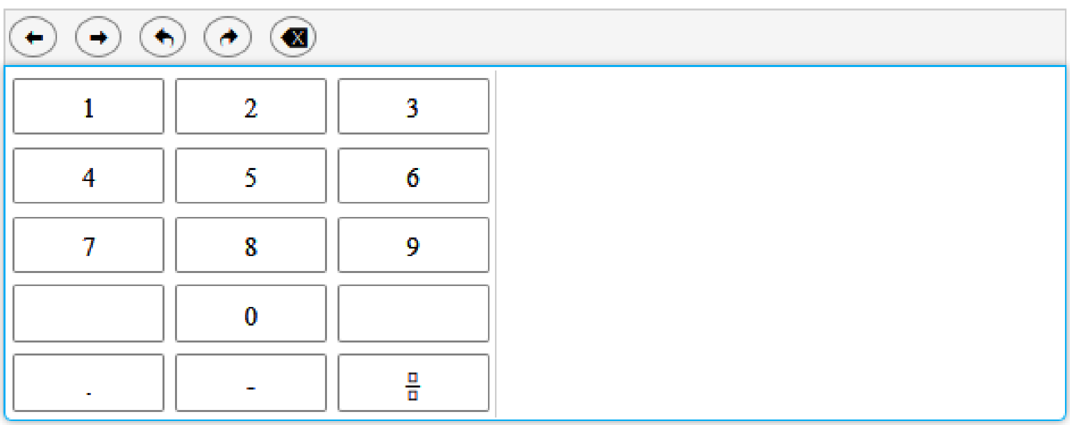
A dilation centered at the origin with a scale factor of 6 is applied to this line.

A. What is the slope of the line after the dilation?

B. What is the value of the y-intercept of the line after the dilation?

$A.$ 12

$B.$ -10.0



Notes on Scoring

This response earns partial credit (1 point) because it shows a correct y-intercept, but an incorrect slope of the line after the dilation.

The student may correctly determine the slope of the original line, 2, but then incorrectly multiplies it by 6 because of the given scale factor.

Sample Response: 1 point

The equation of a line is shown.

$6x - 3y = 5$

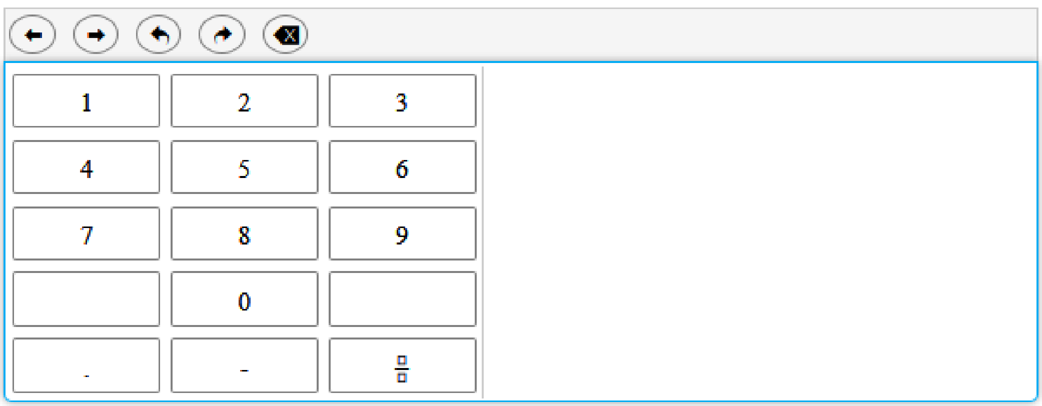
A dilation centered at the origin with a scale factor of 6 is applied to this line.

A. What is the slope of the line after the dilation?

B. What is the value of the y-intercept of the line after the dilation?

$A.$ 2

$B.$ 30



Notes on Scoring

This response earns partial credit (1 point) because it shows a correct slope but an incorrect *y*-intercept of the line after the dilation. The student may understand that if a dilation is applied to a line not passing through the center of dilation, it takes the line to a parallel line. Since parallel lines have equal slopes, the slope of the line after the dilation equals to the slope of the original line, or 2.

The student may also recall that when a line is dilated with the center of dilation at the origin by a scale factor 6, the image of the *y*-intercept remains on the same side of the *y*-axis but 6 times as far from the origin, but forgets to write the given equation in slope-intercept form and mistakenly thinks that 5 is the *y*-intercept. Thus, the student multiplies 5 by 6 to get the incorrect *y*-intercept, 30.

Sample Response: 0 points

The equation of a line is shown.

$6x - 3y = 5$

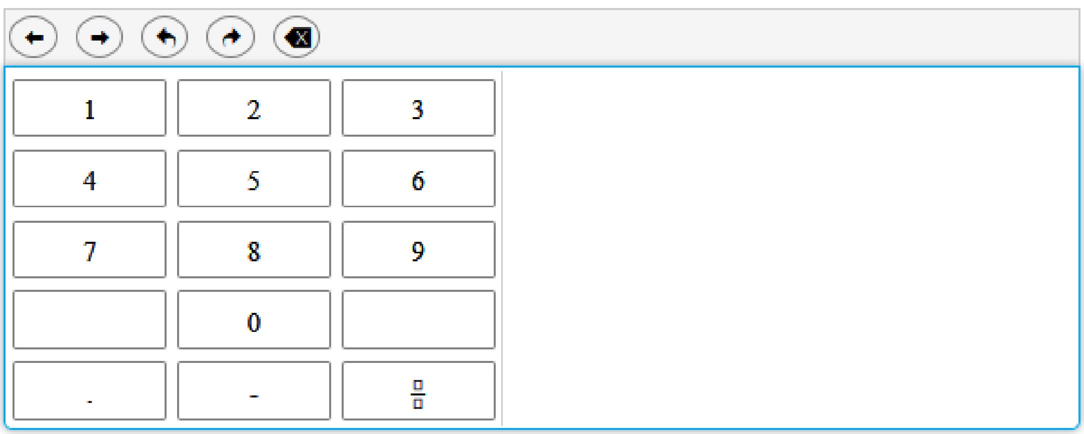
A dilation centered at the origin with a scale factor of 6 is applied to this line.

A. What is the slope of the line after the dilation?

B. What is the value of the y-intercept of the line after the dilation?

A. 36

B. 10



Notes on Scoring

This response earns no credit (0 points) because it shows an incorrect slope and an incorrect y-intercept of the line after the dilation.

Sample Response: 0 points

The equation of a line is shown.

$6x - 3y = 5$

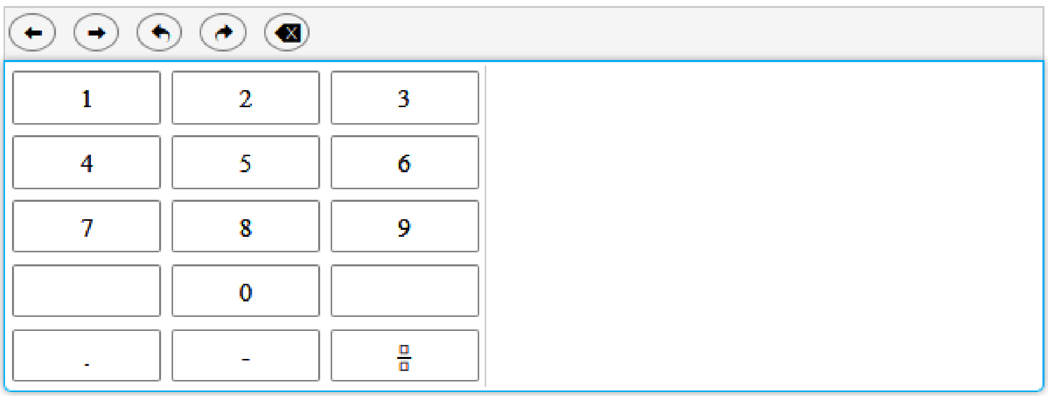
A dilation centered at the origin with a scale factor of 6 is applied to this line.

A. What is the slope of the line after the dilation?

B. What is the value of the y-intercept of the line after the dilation?

$A.$ -2

$B.$ $\frac{30}{3}$



Notes on Scoring

This response earns no credit (0 points) because it shows an incorrect slope and an incorrect *y*-intercept of the line after the dilation. In an attempt to convert the original equation into the slope-intercept form, the student may add $6x$ to both sides of the equation instead of subtracting $6x$ from both sides to get $-3y = 5 + 6x$. Then, he or she may divide both sides by (-3) to get the incorrect equation $y = -2x - \frac{5}{3}$. The student may then identify that since parallel lines have equal slopes, the slope of the line after the dilation equals the slope of the original line, or in this case, -2. Next, the student may apply a scale factor of 6 to the incorrect *y*-intercept $\left(-\frac{5}{3}\right)$ to get -10.

Geometry
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Question 47

Question and Scoring Guidelines

Question 47

Triangle MNO is transformed to produce triangle PQR.

Select all of the transformations that would guarantee triangles MNO and PQR are congruent.

- ☐ a dilation, then a translation
- ☐ a reflection, then a dilation
- ☐ a reflection, then a rotation
- ☐ a rotation, then a translation
- ☐ a translation, then a reflection

Points Possible: 1

Content Cluster: Understand congruence in terms of rigid motions.

Content Standard: Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent. *(G.CO.6)*

Scoring Guidelines

<u>Rationale for First Option:</u> The student may think that if one transformation, a translation, is a rigid motion and another, a dilation, is not a rigid motion then a combination of two transformations remains rigid to produce congruent triangles.

<u>Rationale for Second Option:</u> The student may think that if one transformation, a reflection, is a rigid motion and another, a dilation, is not a rigid motion then a combination of two transformations remains rigid to produce congruent triangles.

<u>Rationale for Third Option:</u> **Key** – The student recognizes that a combination of a reflection and a rotation produces congruent triangles because they are transformations that do not alter the size or shape of a figure.

<u>Rationale for Fourth Option:</u> **Key** – The student recognizes that a combination of a rotation and a translation produces congruent triangles because they are transformations that do not alter the size or shape of a figure.

<u>Rationale for Fifth Option:</u> **Key** – The student recognizes that a combination of a translation and a reflection produces congruent triangles because they are transformations that do not alter the size or shape of a figure.

Sample Response: 1 point

Triangle MNO is transformed to produce triangle PQR.

Select all of the transformations that would guarantee triangles MNO and PQR are congruent.

- [ ] a dilation, then a translation
- [ ] a reflection, then a dilation
- [x] a reflection, then a rotation
- [x] a rotation, then a translation
- [x] a translation, then a reflection

Geometry
Spring 2018 Item Release

Question 48

Question and Scoring Guidelines

Question 48

Rosa collects data on what students at her school like to eat at the movie theater. She asks a random sample of 120 students two questions:

- Do you like to eat popcorn at the movie theater?
- Do you like to eat candy at the movie theater?

Her data are partially shown in the table. Of the students she asks, 60% of those who like to eat popcorn also like to eat candy.

Complete the table to show the number of students in each category.

	Like Popcorn	Don't Like Popcorn	Total
Like Candy			
Don't Like Candy			62
Total	70		120

Points Possible: 1

Content Cluster: Understand independence and conditional probability and use them to interpret data.

Content Standard: Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. *For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results. (S.CP.4)*

Scoring Guidelines

Exemplar Response

	Like Popcorn	Don't Like Popcorn	Total
Like Candy	42	16	58
Don't Like Candy	28	34	62
Total	70	50	120

Other Correct Responses

- Any equivalent decimal values

For this item, a full-credit response includes:

- A correct table (1 point).

Geometry
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Question 48

Sample Responses

Sample Response: 1 point

Rosa collects data on what students at her school like to eat at the movie theater. She asks a random sample of 120 students two questions:

- Do you like to eat popcorn at the movie theater?
- Do you like to eat candy at the movie theater?

Her data are partially shown in the table. Of the students she asks, 60% of those who like to eat popcorn also like to eat candy.

Complete the table to show the number of students in each category.

	Like Popcorn	Don't Like Popcorn	Total
Like Candy	42	16	58
Don't Like Candy	28	34	62
Total	70	50	120

Notes on Scoring

This response earns full credit (1 point) because it shows a completely correct two-way frequency table. Using frequencies given in the table, the first step is to determine the blank subtotals as 120 – 70 = 50 to complete the bottom row and 120 – 62 = 58 to complete the last column.

Next, the value in the top left cell of the table can be determined by using the fact that 60% of those who like to eat popcorn also like to eat candy, or 70 • 0.6 = 42.

Lastly, using all known frequencies, the remaining parts of the table can be completed as:

70 – 42 = 28 (second row, first column)
58 – 42 = 16 (first row, second column)
62 – 28 = 34 (second row, second column)

Sample Response: 1 point

Rosa collects data on what students at her school like to eat at the movie theater. She asks a random sample of 120 students two questions:

- Do you like to eat popcorn at the movie theater?
- Do you like to eat candy at the movie theater?

Her data are partially shown in the table. Of the students she asks, 60% of those who like to eat popcorn also like to eat candy.

Complete the table to show the number of students in each category.

	Like Popcorn	Don't Like Popcorn	Total
Like Candy	42.0	16.0	58.0
Don't Like Candy	28.0	34.0	62
Total	70	50.0	120

Notes on Scoring

This response earns full credit (1 point) because it shows a completely correct two-way frequency table with equivalent entries.

Sample Response: 0 points

Rosa collects data on what students at her school like to eat at the movie theater. She asks a random sample of 120 students two questions:

- Do you like to eat popcorn at the movie theater?
- Do you like to eat candy at the movie theater?

Her data are partially shown in the table. Of the students she asks, 60% of those who like to eat popcorn also like to eat candy.

Complete the table to show the number of students in each category.

	Like Popcorn	Don't Like Popcorn	Total
Like Candy	60	-2	58
Don't Like Candy	10	52	62
Total	70	50	120

Notes on Scoring

This response earns no credit (0 points) because it shows four incorrect entries in the two-way frequency table. First, the student may correctly determine the blank subtotals as 120 – 70 = 50 to complete the bottom row and 120 – 62 = 58 to complete the last column.

Next, the student may incorrectly enter 60 in the top left cell of the table instead of the value representing 60% of those who liked popcorn also liked candy, or 70 • 0.6 = 42. Based on this error, the student completes the rest of the table.

Sample Response: 0 points

Rosa collects data on what students at her school like to eat at the movie theater. She asks a random sample of 120 students two questions:

- Do you like to eat popcorn at the movie theater?
- Do you like to eat candy at the movie theater?

Her data are partially shown in the table. Of the students she asks, 60% of those who like to eat popcorn also like to eat candy.

Complete the table to show the number of students in each category.

	Like Popcorn	Don't Like Popcorn	Total
Like Candy	34.8	23.2	58
Don't Like Candy	35.2	26.8	62
Total	70	50	120

Notes on Scoring

This response earns no credit (0 points) because it shows four incorrect entries in the two-way frequency table. First, the student may correctly determine the blank subtotals as 120 – 70 = 50 to complete the bottom row and 120 – 62 = 58 to complete the last column.

Next, the student may mistakenly calculate 60% of those who liked candy also liked popcorn, or 58 • 0.6 = 34.8, for the top left cell of the table instead of the value representing 60% of those who liked popcorn also liked candy, or 70 • 0.6 = 42. Based on this error, the student completes the rest of the table.

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