

# Ohio's State Tests

**ITEM RELEASE**

**SPRING 2019**

**GEOMETRY**

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# Geometry

## Spring 2019 Item Release

### Content Summary and Answer Key

Question No.*	Item Type	Content Cluster	Content Standard	Depth of Knowledge	Answer Key	Points
1	Multiple Choice Item	Visualize relationships between two-dimensional and three-dimensional objects.	Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects. (G.GMD.4)	Level 1	A	1 point
2	Equation Item	Understand and apply theorems about circles.	Identify and describe relationships among angles, radii, chords, tangents, and arcs and use them to solve problems. Include the relationship between central, inscribed, and circumscribed angles and their intercepted arcs; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle. (G.C.2)	Level 2	---	1 point
4	Equation Item	Experiment with transformations in the plane.	Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another. (G.CO.5)	Level 3	---	1 point
5	Inline Choice Item	Use the rules of probability to compute probabilities of compound events in a uniform probability model.	Apply the Addition Rule, $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$ , and interpret the answer in terms of the model. (S.CP.7)	Level 2	---	1 point

\* The question number matches the item number in the Item Level Report in the Online Reporting System. The items are numbered sequentially in the practice site.

# Geometry

## Spring 2019 Item Release

### Content Summary and Answer Key

Question No.*	Item Type	Content Cluster	Content Standard	Depth of Knowledge	Answer Key	Points
8	Equation Item	Understand independence and conditional probability, and use them to interpret data.	Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results. (S.CP.4)	Level 2	---	1 point
10	Multiple Choice Item	Find arc lengths and areas of sectors of circles.	Find arc lengths and areas of sectors of circles. (G.C.5) a. Apply similarity to relate the length of an arc intercepted by a central angle to the radius. Use the relationship to solve problems.	Level 2	A	1 point
11	Equation Item	Use coordinates to prove simple geometric theorems algebraically and to verify specific geometric statements.	Find the point on a directed line segment between two given points that partitions the segment in a given ratio. (G.GPE.6)	Level 2	---	1 point

\* The question number matches the item number in the Item Level Report in the Online Reporting System. The items are numbered sequentially in the practice site.

# Geometry

## Spring 2019 Item Release

### Content Summary and Answer Key

Question No.*	Item Type	Content Cluster	Content Standard	Depth of Knowledge	Answer Key	Points
12	Gap Match Item	Classify and analyze geometric figures.	Classify two-dimensional figures in a hierarchy based on properties. (G.CO.14)	Level 2	---	1 point
16	Multiple Choice Item	Define trigonometric ratios and solve problems involving right triangles.	Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles. (G.SRT.6)	Level 3	D	1 point
20	Equation Item	Define trigonometric ratios and solve problems involving right triangles.	Explain and use the relationship between the sine and cosine of complementary angles. (G.SRT.7)	Level 2	---	1 point
21	Multiple Choice Item	Make geometric constructions.	Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line. (G.CO.12)	Level 1	D	1 point

\* The question number matches the item number in the Item Level Report in the Online Reporting System. The items are numbered sequentially in the practice site.

# Geometry

## Spring 2019 Item Release

### Content Summary and Answer Key

Question No.*	Item Type	Content Cluster	Content Standard	Depth of Knowledge	Answer Key	Points
28	Multiple Choice Item	Understand independence and conditional probability, and use them to interpret data.	Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer. (S.CP.5)	Level 2	B	1 point
30	Inline Choice Item	Understand the relationships between lengths, area, and volumes.	Understand how and when changes to the measures of a figure (lengths or angles) result in similar and non-similar figures. (G.GMD.5)	Level 2	---	1 point
32	Multiple Choice Item	Use the rules of probability to compute probabilities of compound events in a uniform probability model.	Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and interpret the answer in terms of the model. (S.CP.6)	Level 2	D	1 point
33	Equation Item	Understand similarity in terms of similarity transformations.	Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides. (G.SRT.2)	Level 2	---	2 points

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# Geometry

## Spring 2019 Item Release

### Content Summary and Answer Key

Question No.*	Item Type	Content Cluster	Content Standard	Depth of Knowledge	Answer Key	Points
34	Multiple Choice Item	Understand congruence in terms of rigid motions.	Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions. (G.CO.8)	Level 1	D	1 point
36	Multiple Choice Item	Explain volume formulas and use them to solve problems.	Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. Use dissection arguments, Cavalieri's principle, and informal limit arguments. (G.GMD.1)	Level 3	A	1 point
40	Gap Match Item	Prove geometric theorems both formally and informally using a variety of methods.	Prove and apply theorems about lines and angles. Theorems include but are not restricted to the following: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints. (G.CO.9)	Level 3	---	2 points
41	Equation Item	Apply geometric concepts in modeling situations.	Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios). (G.MG.3)	Level 2	---	1 point

\* The question number matches the item number in the Item Level Report in the Online Reporting System. The items are numbered sequentially in the practice site.



**Geometry**  
**Spring 2019 Item Release**  
**Content Summary and Answer Key**

Question No.*	Item Type	Content Cluster	Content Standard	Depth of Knowledge	Answer Key	Points
49	Equation Item	Understand the relationships between lengths, area, and volumes.	When figures are similar, understand and apply the fact that when a figure is scaled by a factor of $k$ , the effect on lengths, areas, and volumes is that they are multiplied by $k$ , $k^2$ , and $k^3$ , respectively. (G.GMD.6)	Level 2	---	1 point

*\* The question number matches the item number in the Item Level Report in the Online Reporting System. The items are numbered sequentially in the practice site.*

## Depth of Knowledge (DOK)

DOK refers to the complexity of thinking required to complete a task in a given item. Items with a DOK 1 designation focus on the recall of information, such as definitions and terms, and simple procedures. Items with a DOK 2 designation require students to make decisions, solve routine problems, perform calculations, or recognize patterns. Items with a DOK 3 designation feature higher-order cognitive tasks. These DOK 3 tasks include but are not limited to: critiquing a statement and forming a conclusion; explaining, justifying, or proving a statement; or approaching abstract, complex, open-ended, and non-routine problems. Each grade's blueprint contains information about the number of points of opportunity students will encounter at each DOK level.

**Table 1: Math Descriptors – Applying Depth of Knowledge Levels for Mathematics (Webb, 2002) & NAEP 2002 Mathematics Levels of Complexity**  
(M. Petit, Center for Assessment 2003, K. Hess, Center for Assessment, updated 2006)

Level 1 Recall	Level 2 Skills/Concepts	Level 3 Strategic Thinking	Level 4 Extended Thinking
a. Recall, observe, or recognize a fact, definition, term, or property b. Apply/compute a well-known algorithm (e.g., sum, quotient) c. Apply a formula d. Determine the area or perimeter of rectangles or triangles given a drawing and labels e. Identify a plane or three-dimensional figure f. Measure g. Perform a specified or routine procedure (e.g., apply rules for rounding) h. Evaluate an expression i. Solve a one-step word problem j. Retrieve information from a table or graph	a. Classify plane and three-dimensional figures b. Interpret information from a simple graph c. Use models to represent mathematical concepts d. <b>Solve a routine problem</b> requiring multiple steps/ decision points, or the application of multiple concepts e. Compare and/or contrast figures or statements f. Construct 2-dimensional patterns for 3-dimensional models, such as cylinders and cones g. Provide justifications for steps in a solution process h. Extend a pattern	a. Interpret information from a complex graph b. Explain thinking when more than one response is possible c. Make and/or justify conjectures d. Use evidence to develop logical arguments for a concept e. Use concepts to solve non-routine problems f. Perform procedures with multiple steps and multiple decision points g. Generalize a pattern h. Describe, compare, and contrast solution methods i. Formulate a mathematical model for a complex situation j. Provide mathematical justifications	a. Relate mathematical concepts to other content areas b. Relate mathematical concepts to real-world applications in new situations c. Apply a mathematical model to illuminate a problem, situation d. Conduct a project that specifies a problem, identifies solution paths, solves the problem, and reports results e. Design a mathematical model to inform and solve a practical or abstract situation f. Develop generalizations of the results obtained and the strategies used and apply them to new problem situations

**Table 1** continued on next page.

Level 1 Recall	Level 2 Skills/Concepts	Level 3 Strategic Thinking	Level 4 Extended Thinking
k. Recall, identify, or make conversions between and among representations or numbers (fractions, decimals, and percents), or within and between customary and metric measures l. Locate numbers on a number line, or points on a coordinate grid m. Solve linear equations n. Represent math relationships in words, pictures, or symbols o. Read, write, and compare decimals in scientific notation	i. Retrieve information from a table, graph, or figure and use it to solve a problem requiring multiple steps j. Translate between tables, graphs, words and symbolic notation k. Make direct translations between problem situations and symbolic notation l. Select a procedure according to criteria and perform it m. Specify and explain relationships between facts, terms, properties, or operations n. Compare, classify, organize, estimate, or order data	k. Solve a multiple-step problem and provide support with a mathematical explanation that justifies the answer l. Solve 2-step linear equations/inequalities in one variable over the rational numbers, interpret solution(s) in the original context, and verify reasonableness of results m. Translate between a problem situation and symbolic notation that is not a direct translation n. Formulate an original problem, given a situation o. Analyze the similarities and differences between procedures p. Draw conclusion from observations or data, citing evidence	g. Apply one approach among many to solve problems h. Apply understanding in a novel way, providing an argument/justification for the application  <b>NOTE:</b> Level 4 involves such things as complex restructuring of data or establishing and evaluating criteria to solve problems.

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**Geometry**  
**Spring 2019 Item Release**

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**Question 1**

**Question and Scoring Guidelines**

## Question 1

A cylinder is sliced vertically along a dotted line, as shown.



Which two-dimensional shape is created from this cross section?

(A)



(C)



(B)



(D)



**Points Possible:** 1

**Content Cluster:** Visualize relationships between two-dimensional and three-dimensional objects.

**Content Standard:** Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects. (G.GMD.4)

**Depth of Knowledge:** Level 1

- e. Identify a plane or three-dimensional figure
- j. Retrieve information from a table or graph

## Scoring Guidelines

Rationale for Option A: **Key** – The student correctly notes the result of a rotation about a vertical line through its center is a cylinder. Thus, a vertical slice of a cylinder along the dotted line leads to a rectangular cross section.

Rationale for Option B: This is incorrect. The student may think that since the sides of the cylinder are straight and the bases of the cylinder are circles, that the cross section has 2 straight sides and 2 curved sides. However, if this shape is rotated around a vertical line around its center, the result of the rotation is a cylinder-like figure with bowed out sides. Thus, this would be a vertical cross section for a cylinder-like figure with bowed out sides.

Rationale for Option C: This is incorrect. The student may think that every cross section of a cylinder is a circle, but instead, this is only true for horizontal slices parallel to the base.

Rationale for Option D: This is incorrect. The student may think that a cross section of a cylinder would have to be some sort of curved shape and also notice that the cylinder is taller than it is wide.

Sample Response: 1 point

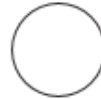
A cylinder is sliced vertically along a dotted line, as shown.



Which two-dimensional shape is created from this cross section?



C



B



D



**Geometry**  
**Spring 2019 Item Release**

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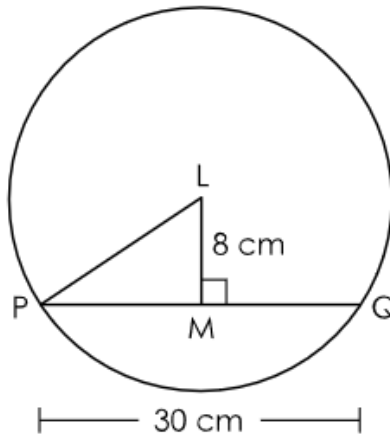
**Question 2**

**Question and Scoring Guidelines**



## Question 2

In the figure shown,  $L$  is the center of the circle and  $\overline{PQ}$  is a chord of the circle measuring 30 centimeters (cm).



What is the length, in centimeters, of  $\overline{PL}$ ?

 *cm*

<div> <div>←</div> <div>→</div> <div>↶</div> <div>↷</div> <div>✖</div> </div>															
1	2	3	+	-	•	÷									
4	5	6	<	≤	=	≥	>								
7	8	9	$\frac{\square}{\square}$	$\square^\square$	$\square_\square$	( )		$\sqrt{\square}$	$\sqrt[\square]{\square}$	$\pi$	$i$				
0	.	-	sin	cos	tan	arcsin	arccos	arctan							

**Points Possible:** 1

**Content Cluster:** Understand and apply theorems about circles.

**Content Standard:** Identify and describe relationships among angles, radii, chords, tangents, and arcs and use them to solve problems. Include the relationship between central, inscribed, and circumscribed angles and their intercepted arcs; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle. (G.C.2)

**Depth of Knowledge:** Level 2

- b. Interpret information from a simple graph
- i. Retrieve information from a table, graph, or figure and use it solve a problem requiring multiple steps

## Scoring Guidelines

### Exemplar Response

- 17

### Other Correct Responses

- any equivalent value

For the item, a full-credit response includes

- a correct value (1 point).

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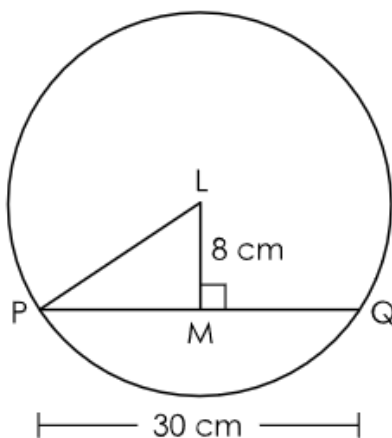
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**Question 2**

**Sample Responses**

Sample Response: 1 point

In the figure shown,  $L$  is the center of the circle and  $\overline{PQ}$  is a chord of the circle measuring 30 centimeters (cm).



What is the length, in centimeters, of  $\overline{PL}$ ?

17

cm

←	→	↶	↷	✖										
1	2	3	+	-	•	÷								
4	5	6	<	≤	=	≥	>							
7	8	9	$\frac{\square}{\square}$	$\square^\square$	$\square_\square$	( )		$\sqrt{\square}$	$\sqrt[n]{\square}$	$\pi$	$i$			
0	.	-	sin	cos	tan	arcsin	arccos	arctan						

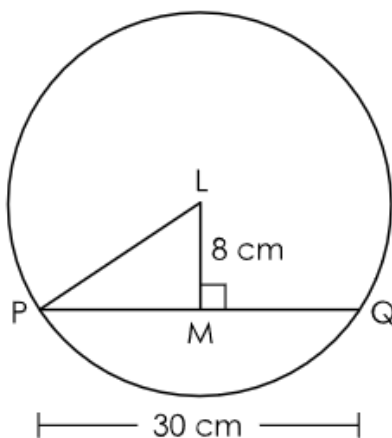
### Notes on Scoring

This response earns full credit (1 point) because it shows the correct length of the radius,  $\overline{PL}$ , of the given circle.

Per the relationship between the radii and the chords of a circle, a radius that is perpendicular to a chord bisects the chord. If  $\overline{LM}$  is extended, it becomes a radius of the circle that is perpendicular to  $\overline{PQ}$ , and the length of  $\overline{PM}$  is 15 cm because the radius  $\overline{LM}$  divides the chord  $\overline{PQ}$ , that is 30 cm long, in half. The triangle  $PML$  is a right triangle with legs of 8 cm and 15 cm long. The length of the hypotenuse  $\overline{PL}$  can be calculated using the Pythagorean Theorem, or  $\overline{PL} = \sqrt{15^2 + 8^2} = \sqrt{289} = 17$ .

**Sample Response: 1 point**

In the figure shown, L is the center of the circle and  $\overline{PQ}$  is a chord of the circle measuring 30 centimeters (cm).



What is the length, in centimeters, of  $\overline{PL}$ ?

$$\sqrt{15^2 + 8^2}$$

cm

<div> <div>←</div> <div>→</div> <div>↶</div> <div>↷</div> <div>✖</div> </div>														
1	2	3	+	-	•	÷								
4	5	6	<	≤	=	≥	>							
7	8	9	$\frac{\Box}{\Box}$	$\Box^\Box$	$\Box_\Box$	( )		$\sqrt{\Box}$	$\sqrt[\Box]{\Box}$	$\pi$	$i$			
0	.	-	sin	cos	tan	arcsin	arccos	arctan						

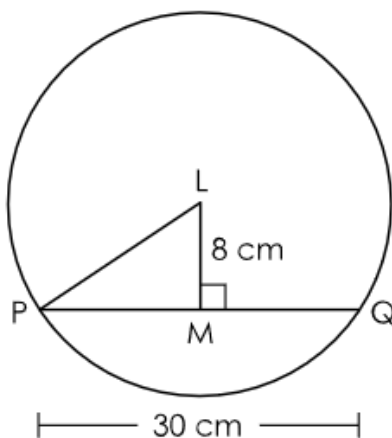
**Notes on Scoring**

This response earns full credit (1 point) because it shows the correct length of the radius,  $\overline{PL}$ , of the given circle in equivalent form.

The student demonstrates the correct use of the Pythagorean Theorem to find the length of  $\overline{PL}$  to be 17 but does not evaluate the square root.

**Sample Response: 0 points**

In the figure shown,  $L$  is the center of the circle and  $\overline{PQ}$  is a chord of the circle measuring 30 centimeters (cm).



What is the length, in centimeters, of  $\overline{PL}$ ?

289

cm

<div>← → ↶ ↷ ✕</div>														
1	2	3	+	-	•	÷								
4	5	6	<	≤	=	≥	>							
7	8	9	$\frac{\Box}{\Box}$	$\Box^\Box$	$\Box_\Box$	( )		$\sqrt{\Box}$	$\sqrt[\Box]{\Box}$	$\pi$	$i$			
0	.	-	sin	cos	tan	arcsin	arccos	arctan						

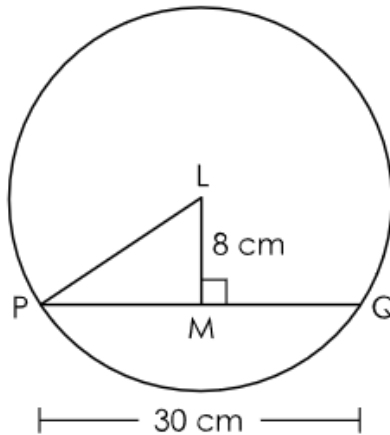
**Notes on Scoring**

This response earns no credit (0 points) because it shows an incorrect length of the radius,  $\overline{PL}$ , of the given circle.

The student demonstrates the correct use of the Pythagorean Theorem to find the length of  $\overline{PL}$  as  $\sqrt{(15^2+8^2)} = \sqrt{289}$  but forgets to apply the square root to 289.

**Sample Response: 0 points**

In the figure shown,  $L$  is the center of the circle and  $\overline{PQ}$  is a chord of the circle measuring 30 centimeters (cm).



What is the length, in centimeters, of  $\overline{PL}$ ?

31.04

cm

←	→	↶	↷	✖										
1	2	3	+	-	•	÷								
4	5	6	<	≤	=	≥	>							
7	8	9	$\frac{\square}{\square}$	$\square^\square$	$\square_\square$	( )		$\sqrt{\square}$	$\sqrt[n]{\square}$	$\pi$	$i$			
0	.	-	sin	cos	tan	arcsin	arccos	arctan						

**Notes on Scoring**

This response earns no credit (0 points) because it shows an incorrect length of the radius,  $\overline{PL}$ , of the given circle.

The student does not realize that if  $\overline{LM}$  is perpendicular to a chord  $\overline{PQ}$ , then  $\overline{LM}$  bisects the chord, so  $PM = 15$ . The student applies the Pythagorean Theorem but uses an incorrect value, 30, instead of 15, to get  $\sqrt{(30^2 + 8^2)} = \sqrt{964} \approx 31.04$ .



**Geometry**  
**Spring 2019 Item Release**

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**Question 4**

**Question and Scoring Guidelines**

## Question 4

A sequence of translations maps  $\triangle GHI$  to  $\triangle G'H'I'$ .

- $\triangle GHI$  has vertices at  $G(-8, 2)$ ,  $H(13, 2)$ , and  $I(-2, 10)$ .
- The coordinates of  $G'$  are  $(-1, -3)$ .

What are the coordinates for  $H'$  and  $I'$ ?

$H'(\text{ }, \text{ })$

$I'(\text{ }, \text{ })$

1	2	3	+	-	•	÷					
4	5	6	<	≤	=	≥	>				
7	8	9	$\frac{\square}{\square}$	$\square^\square$	$\square_\square$	( )		$\sqrt{\square}$	$\sqrt[\square]{\square}$	$\pi$	$i$
0	.	-	sin	cos	tan	arcsin	arccos	arctan			

**Points Possible:** 1

**Content Cluster:** Experiment with transformations in the plane.

**Content Standard:** Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another. (G.CO.5)

**Depth of Knowledge:** Level 3

- c. Make and/or justify conjectures
- g. Generalize a pattern
- m. Translate between a problem situation and symbolic notation that is not a direct translation

## Scoring Guidelines

### Exemplar Response

- $H' (20, -3)$   
 $I' (5, 5)$

### Other Correct Responses

- any equivalent value

For the item, a full-credit response includes

- a correct set of points (1 point).

**Geometry**  
**Spring 2019 Item Release**

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**Question 4**

**Sample Responses**

**Sample Response: 1 point**

A sequence of translations maps  $\triangle GHI$  to  $\triangle G'H'I'$ .

- $\triangle GHI$  has vertices at  $G(-8, 2)$ ,  $H(13, 2)$ , and  $I(-2, 10)$ .
- The coordinates of  $G'$  are  $(-1, -3)$ .

What are the coordinates for  $H'$  and  $I'$ ?

$H' (20, -3)$

$I' (5, 5)$

<div><div>←</div><div>→</div><div>↶</div><div>↷</div><div>✖</div></div>														
1	2	3	+	-	•	÷								
4	5	6	<	≤	=	≥	>							
7	8	9	$\frac{\square}{\square}$	$\square^\square$	$\square_\square$	( )		$\sqrt{\square}$	$\sqrt[\square]{\square}$	$\pi$	$i$			
0	.	-	sin	cos	tan	arcsin	arccos	arctan						

**Notes on Scoring**

This response earns full credit (1 point) because it shows two correct pairs of coordinates for the vertices of the triangle  $G'H'I'$ .

First, establish the rule for the transformation using the fact that the coordinates of the two corresponding vertices are  $G(-8, 2)$  and  $G'(-1, -3)$ .

The transformation carries all points of the original triangle  $GHI$  by the same rule as it carries a vertex  $G$  to a vertex  $G'$ . Since  $-8 + 7 = -1$  and  $2 - 5 = -3$ , the transformation can be described by the rule:  $(x, y) \rightarrow (x + 7, y - 5)$ . Apply this rule to the vertices  $H$  and  $I$ .

The coordinates of  $H'$  are  $(13 + 7; 2 - 5)$  or  $(20, -3)$ , and the coordinates of  $I'$  are  $(-2 + 7, 10 - 5)$  or  $(5, 5)$ .

### Sample Response: 1 point

A sequence of translations maps  $\triangle GHI$  to  $\triangle G'H'I'$ .

- $\triangle GHI$  has vertices at  $G(-8, 2)$ ,  $H(13, 2)$ , and  $I(-2, 10)$ .
- The coordinates of  $G'$  are  $(-1, -3)$ .

What are the coordinates for  $H'$  and  $I'$ ?

$$H'(13 + 7, 2 - 5)$$

$$I'(-2 + 7, 10 - 5)$$

$\leftarrow$	$\rightarrow$	$\curvearrowleft$	$\curvearrowright$	$\times$										
1	2	3	+	-	$\cdot$	$\div$								
4	5	6	<	$\leq$	=	$\geq$	>							
7	8	9	$\frac{\square}{\square}$	$\square^\square$	$\square_\square$	( )		$\sqrt{\square}$	$\sqrt[\square]{\square}$	$\pi$	$i$			
0	.	-	sin	cos	tan	arcsin	arccos	arctan						

#### Notes on Scoring

This response earns full credit (1 point) because it shows two correct pairs of coordinates for the vertices of the triangle  $G'H'I'$ .

The student correctly identifies that point  $G$  is translated 7 units to the right and 5 units down and applies the same rule to points  $H$  and  $I$  but leaves calculations incomplete.

### Sample Response: 0 points

A sequence of translations maps  $\triangle GHI$  to  $\triangle G'H'I'$ .

- $\triangle GHI$  has vertices at  $G(-8, 2)$ ,  $H(13, 2)$ , and  $I(-2, 10)$ .
- The coordinates of  $G'$  are  $(-1, -3)$ .

What are the coordinates for  $H'$  and  $I'$ ?

$H'(6, 7)$   
 $I'(-9, 15)$

$\leftarrow$	$\rightarrow$	$\curvearrowleft$	$\curvearrowright$	$\times$										
1	2	3	+	-	$\bullet$	$\div$								
4	5	6	<	$\leq$	=	$\geq$	>							
7	8	9	$\frac{\square}{\square}$	$\square^\square$	$\square_\square$	( )		$\sqrt{\square}$	$\sqrt[\square]{\square}$	$\pi$	$i$			
0	.	-	sin	cos	tan	arcsin	arccos	arctan						

#### Notes on Scoring

This response earns no credit (0 points) because it shows two incorrect pairs of coordinates for the vertices of the triangle  $GHI$ .

The student may find the incorrect rule for the translation from point  $G'$  to point  $G$  as  $(x, y) \rightarrow (x - 7, y + 5)$ , instead of from point  $G$  to point  $G'$ . Under this rule, the coordinates of  $H'$  are  $(13 - 7; 2 + 5)$  or  $(6, 7)$ , and the coordinates of  $I'$  are  $(-2 - 7, 10 + 5)$  or  $(-9, 15)$ .



### Sample Response: 0 points

A sequence of translations maps  $\triangle GHI$  to  $\triangle G'H'I'$ .

- $\triangle GHI$  has vertices at  $G(-8, 2)$ ,  $H(13, 2)$ , and  $I(-2, 10)$ .
- The coordinates of  $G'$  are  $(-1, -3)$ .

What are the coordinates for  $H'$  and  $I'$ ?

$H' (6, -3)$   
 $I' (-9, 5)$

$\leftarrow$	$\rightarrow$	$\curvearrowleft$	$\curvearrowright$	$\times$										
1	2	3	+	-	$\cdot$	$\div$								
4	5	6	<	$\leq$	=	$\geq$	>							
7	8	9	$\frac{\square}{\square}$	$\square^\square$	$\square_\square$	( )		$\sqrt{\square}$	$\sqrt[\square]{\square}$	$\pi$	$i$			
0	.	-	sin	cos	tan	arcsin	arccos	arctan						

#### Notes on Scoring

This response earns no credit (0 points) because it shows two incorrect pairs of coordinates for the vertices of the triangle's image.

The student may find the incorrect rule for the translation from point  $G$  to point  $G'$  as  $(x, y) \rightarrow (x - 7, y - 5)$  instead of  $(x, y) \rightarrow (x + 7, y - 5)$ . Under this rule, the coordinates of  $H'$  are  $(13 - 7, 2 - 5)$  or  $(6, -3)$ , and the coordinates of  $I'$  are  $(-2 - 7, 10 - 5)$  or  $(-9, 5)$ .

**Geometry**  
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**Question 5**

**Question and Scoring Guidelines**

## Question 5

A soccer coach determines that there is a 50% chance that a star player, Ralph, will play in a tournament.

- The probability that another star player, Dan, will play is 0.48.
- The probability that both Ralph and Dan will play in the tournament is 0.25.

Select phrases to complete the statement.

To find the probability that either Ralph or Dan will play in the tournament, first add  and then .

### Drop down choices

To find the probability that either Ralph or Dan will play in the tournament, first add  and then .

0.50 and 0.48  
0.50 and 0.25  
0.48 and 0.25

subtract 0.25 from the sum.  
multiply the sum by 0.48.  
divide 0.50 by the sum.  
subtract 0.50 from the sum.  
multiply the sum by 0.25.  
divide 0.48 by the sum.

**Points Possible:** 1

**Content Cluster:** Use the rules of probability to compute probabilities of compound events in a uniform probability model.

**Content Standard:** Apply the Addition Rule,  $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$ , and interpret the answer in terms of the model.  
(S.CP.7)

**Depth of Knowledge:** Level 2

- c. Use models to represent mathematical concepts
- k. Make direct translations between problem situations and symbolic notation
- l. Select a procedure according to criteria and perform it

## Scoring Guidelines

### Exemplar Response

- “0.50 and 0.48” and “subtract 0.25 from the sum”

### Other Correct Responses

- N/A

For the item, a full-credit response includes

- the correctly completed statement (1 point).

**Geometry**  
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**Question 5**

**Sample Responses**

## Sample Response: 1 point

A soccer coach determines that there is a 50% chance that a star player, Ralph, will play in a tournament.

- The probability that another star player, Dan, will play is 0.48.
- The probability that both Ralph and Dan will play in the tournament is 0.25.

Select phrases to complete the statement.

To find the probability that either Ralph or Dan will play in the tournament, first add  and then .

### Notes on Scoring

This response earns full credit (1 point) because it shows the correctly completed statement using probabilities of events.

To find a probability of either Ralph or Dan playing in the tournament, use the Addition Rule,  $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$ , where  $P(A)$  is a probability of one event,  $P(B)$  is the probability of another event and  $P(A \text{ and } B)$  is the probability of both events happening at the same time.

If there is a 50% chance that Ralph plays in the tournament, then the probability of him playing is 0.5, or  $P(A) = 0.5$ . The probability that Dan plays is 0.48, or  $P(B) = 0.48$ , and the probability that both Ralph and Dan play is 0.25, or  $P(A \text{ and } B)$ . Following the Addition Rule, the probability that either Ralph or Dan plays in the tournament, first add 0.50 and 0.48, and then subtract 0.25 from the sum.

### Sample Response: 0 points

A soccer coach determines that there is a 50% chance that a star player, Ralph, will play in a tournament.

- The probability that another star player, Dan, will play is 0.48.
- The probability that both Ralph and Dan will play in the tournament is 0.25.

Select phrases to complete the statement.

To find the probability that either Ralph or Dan will play in the tournament, first add  and then .

#### Notes on Scoring

This response earns no credit (0 points) because it shows an incorrectly completed statement using probabilities of events.

To find a probability of either Ralph or Dan playing in the tournament, the student may incorrectly use the Addition Rule,  $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$ , by first adding  $P(A) + P(B)$ , or 0.5 and 0.48, and then multiplying the result by  $P(A) + P(B)$ , or 0.25, instead of subtracting 0.25.

### Sample Response: 0 points

A soccer coach determines that there is a 50% chance that a star player, Ralph, will play in a tournament.

- The probability that another star player, Dan, will play is 0.48.
- The probability that both Ralph and Dan will play in the tournament is 0.25.

Select phrases to complete the statement.

To find the probability that either Ralph or Dan will play in the tournament, first add  and then .

#### Notes on Scoring

This response earns no credit (0 points) because it shows an incorrectly completed statement using probabilities of events.

The student may attempt to use the Addition Rule,  $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$ , but incorrectly use two smaller probabilities, 0.48 and 0.25, to replace  $P(A) + P(B)$  and one larger probability, 0.50, to replace  $P(A \text{ and } B)$ , so that the statement reads “first add 0.48 and 0.25, and then subtract 0.50 from the sum”.



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**Question 8**

**Question and Scoring Guidelines**

## Question 8

A survey was conducted to determine whether a group of 11th graders and 12th graders preferred to go to the amusement park or to the zoo for a class trip. The results are shown in the table.

	Amusement Park	Zoo
11th Graders	32	18
12th Graders	24	26

Based on the table, what is the probability that a student preferred a class trip to the zoo given they are in 11th grade?

1

2

3

4

5

6

7

8

9

0

.

-

$\frac{\Box}{\Box}$

**Points Possible:** 1

**Content Cluster:** Understand independence and conditional probability, and use them to interpret data.

**Content Standard:** Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results. (*S.CP.4*)

**Depth of Knowledge:** Level 2

- i. Retrieve information from a table, graph, or figure and use it to solve a problem requiring multiple steps
- k. Make direct translations between problem situations and symbolic notation

## Scoring Guidelines

### Exemplar Response

- 0.36

### Other Correct Responses

- any equivalent value

For the item, a full-credit response includes

- the correct value (1 point).

**Geometry**  
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**Question 8**

**Sample Responses**

**Sample Response: 1 point**

A survey was conducted to determine whether a group of 11th graders and 12th graders preferred to go to the amusement park or to the zoo for a class trip. The results are shown in the table.

	Amusement Park	Zoo
11th Graders	32	18
12th Graders	24	26

Based on the table, what is the probability that a student preferred a class trip to the zoo given they are in 11th grade?

0.36



1	2	3
4	5	6
7	8	9
	0	
.	-	$\frac{\Box}{\Box}$

**Notes on Scoring**

This response earns full credit (1 point) because it shows the correct conditional probability of an 11<sup>th</sup> grade student preferring the zoo.

The situation requires the ability to interpret information summarized in a two-way table. The total number of 11<sup>th</sup> graders is  $32 + 18$  or 50 students. Out of these, 18 students prefer to go to the zoo. Therefore, the probability that the student prefers to go to the zoo, given that the student is an 11<sup>th</sup> grader is  $\frac{18}{50}$ , or 0.36.

### Sample Response: 1 point

A survey was conducted to determine whether a group of 11th graders and 12th graders preferred to go to the amusement park or to the zoo for a class trip. The results are shown in the table.

	Amusement Park	Zoo
11th Graders	32	18
12th Graders	24	26

Based on the table, what is the probability that a student preferred a class trip to the zoo given they are in 11th grade?

$$\frac{18}{50}$$



1	2	3
4	5	6
7	8	9
	0	
.	-	$\frac{\Box}{\Box}$

**Notes on Scoring**

This response earns full credit (1 point) because it shows the correct conditional probability of an 11<sup>th</sup> grade student preferring the zoo.

The situation requires the ability to interpret information summarized in a two-way table. The total number of 11<sup>th</sup> graders is  $32 + 18$  or 50 students. Out of these, 18 students prefer to go to the zoo. Therefore, the probability that the student prefers to go to the zoo, given that the student is an 11<sup>th</sup> grader is  $\frac{18}{50}$ .



### Sample Response: 1 point

A survey was conducted to determine whether a group of 11th graders and 12th graders preferred to go to the amusement park or to the zoo for a class trip. The results are shown in the table.

	Amusement Park	Zoo
11th Graders	32	18
12th Graders	24	26

Based on the table, what is the probability that a student preferred a class trip to the zoo given they are in 11th grade?

$$\frac{9}{25}$$



1	2	3
4	5	6
7	8	9
	0	
.	-	$\frac{\Box}{\Box}$

#### Notes on Scoring

This response earns full credit (1 point) because it shows the correct conditional probability of an 11<sup>th</sup> grade student preferring the zoo in the equivalent form.

The probability that the student prefers to go to the zoo, given that the student is an 11<sup>th</sup> grader is  $\frac{18}{50}$ , or  $\frac{9}{25}$ .

### Sample Response: 0 points

A survey was conducted to determine whether a group of 11th graders and 12th graders preferred to go to the amusement park or to the zoo for a class trip. The results are shown in the table.

	Amusement Park	Zoo
11th Graders	32	18
12th Graders	24	26

Based on the table, what is the probability that a student preferred a class trip to the zoo given they are in 11th grade?

$$\frac{9}{16}$$



1	2	3
4	5	6
7	8	9
	0	
.	-	$\frac{\Box}{\Box}$

#### Notes on Scoring

This response earns no credit (0 points) because it shows an incorrect conditional probability of an 11<sup>th</sup> grade student preferring the zoo.

The student may incorrectly calculate the ratio of the number of 11<sup>th</sup> grade students who prefer the zoo to the number of 11<sup>th</sup> grade students who prefer the amusement park,  $\frac{18}{32}$  or  $\frac{9}{16}$ , instead of calculating the probability of an 11<sup>th</sup> grade student preferring the zoo, or  $\frac{18}{50}$ .

### Sample Response: 0 points

A survey was conducted to determine whether a group of 11th graders and 12th graders preferred to go to the amusement park or to the zoo for a class trip. The results are shown in the table.

	Amusement Park	Zoo
11th Graders	32	18
12th Graders	24	26

Based on the table, what is the probability that a student preferred a class trip to the zoo given they are in 11th grade?

$$\frac{18}{44}$$



1	2	3
4	5	6
7	8	9
	0	
.	-	$\frac{\Box}{\Box}$

#### Notes on Scoring

This response earns no credit (0 points) because it shows an incorrect conditional probability of an 11<sup>th</sup> grade student preferring the zoo.

Instead, the student may calculate the probability of an 11<sup>th</sup> grader preferring a class trip to the zoo out of all students who prefer to go to the zoo, or  $\frac{18}{18+26} = \frac{18}{44}$ .

**Geometry**  
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**Question 10**

**Question and Scoring Guidelines**

## Question 10

A circle with center L contains points J and K. Circle L is dilated by a factor of 2, resulting in a new circle with center P. Points M and N are on circle P such that central angle MPN has the same measure as central angle JLK.

Which statement correctly identifies the relationship between the arc length of JK and the arc length of MN?

- Ⓐ The arc length of JK is half the arc length of MN.
- Ⓑ The arc length of MN is half the arc length of JK.
- Ⓒ The arc length of JK is a quarter of the arc length of MN.
- Ⓓ The arc length of MN is a quarter of the arc length of JK.

**Points Possible:** 1

**Content Cluster:** Find arc lengths and areas of sectors of circles.

**Content Standard:** Find arc lengths and areas of sectors of circles.  
(G.C.5)

a. Apply similarity to relate the length of an arc intercepted by a central angle to the radius. Use the relationship to solve problems.

**Depth of Knowledge:** Level 2

- c. Use models to represent mathematical concepts
- e. Compare and/or contrast figures or statements

## Scoring Guidelines

Rationale for Option A: Key – The student understands that each arc length is proportional to the radius (Arc Length = (measure of central angle) (radius)). So, after the dilation by a factor 2, the radius of circle  $P$  is 2 times the radius of circle  $L$ , and for the same central angle, the longer arc is 2 times as long as the shorter arc length, and the shorter arc length is  $\frac{1}{2}$  that of the longer arc length.

Rationale for Option B: This is incorrect. The student may understand that arc length is proportional to the radius but switched the order of the proportion.

Rationale for Option C: This is incorrect. The student may confuse the arc length with the area of a sector and think that if the radius of a circle  $L$  is multiplied by 2, then the area of a sector is going to be multiplied by  $2^2$  or 4. Therefore, if  $4 \cdot$  the arc length of  $JK$  = the arc length of  $MN$ , then the arc length of  $JK = \frac{1}{4} \cdot$  the arc length of  $MN$ .

Rationale for Option D: This is incorrect. The student may confuse circle  $L$  with circle  $P$  and the arc length with the area of a sector. The student may incorrectly think that if the radius of a circle  $P$  is multiplied by 2, then the area of a sector is going to be multiplied by  $2^2$  or 4. Therefore, if  $4 \cdot$  the arc length of  $MN$  = the arc length of  $JK$ , then the arc length of  $MN = \frac{1}{4} \cdot$  the arc length of  $JK$ .

### Sample Response: 1 point

A circle with center  $L$  contains points  $J$  and  $K$ . Circle  $L$  is dilated by a factor of 2, resulting in a new circle with center  $P$ . Points  $M$  and  $N$  are on circle  $P$  such that central angle  $MPN$  has the same measure as central angle  $JKL$ .

Which statement correctly identifies the relationship between the arc length of  $JK$  and the arc length of  $MN$ ?

- ☒ The arc length of  $JK$  is half the arc length of  $MN$ .
- ☐ The arc length of  $MN$  is half the arc length of  $JK$ .
- ☐ The arc length of  $JK$  is a quarter of the arc length of  $MN$ .
- ☐ The arc length of  $MN$  is a quarter of the arc length of  $JK$ .

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**Question 11**


**Question and Scoring Guidelines**

## Question 11

Point A is located at  $(-1, -5)$ . The midpoint of line segment AB is point C  $(2, 3)$ .

What are the coordinates of point B?

(  ,  )

				
1	2	3		
4	5	6		
7	8	9		
	0			
.	-			

**Points Possible:** 1

**Content Cluster:** Use coordinates to prove simple geometric theorems algebraically and to verify specific geometric statements.

**Content Standard:** Find the point on a directed line segment between two given points that partitions the segment in a given ratio. (G.GPE.6)

**Depth of Knowledge:** Level 2

- e. Compare and/or contrast figures or statements
- l. Select a procedure according to criteria and perform it



# Scoring Guidelines

## Exemplar Response

- $(5, 11)$

## Other Correct Responses

- any equivalent decimal values

For the item, a full-credit response includes

- a correct ordered pair (1 point).

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**Question 11**





**Sample Responses**

**Sample Response: 1 point**

Point A is located at  $(-1, -5)$ . The midpoint of line segment AB is point C  $(2, 3)$ .

What are the coordinates of point B?

(, )

				
<input type="text" value="1"/>	<input type="text" value="2"/>	<input type="text" value="3"/>		
<input type="text" value="4"/>	<input type="text" value="5"/>	<input type="text" value="6"/>		
<input type="text" value="7"/>	<input type="text" value="8"/>	<input type="text" value="9"/>		
<input type="text"/>	<input type="text" value="0"/>	<input type="text"/>		
<input type="text" value="."/>	<input type="text" value="-"/>	<input type="text" value="÷"/>		

### Notes on Scoring

This response earns full credit (1 point) because it shows correctly identified coordinates of the point.

There are several ways to answer this question.

One of them is to use the Midpoint formula. The midpoint of the line segment with endpoints  $(x_1, y_1)$  and  $(x_2, y_2)$  is the point with coordinates  $(x, y)$ , where  $x = \frac{x_1 + x_2}{2}$  and  $y = \frac{y_1 + y_2}{2}$ . Since in this situation the coordinates of the endpoint A are  $(-1, -5)$  and the coordinates of the midpoint of a line segment AB are  $(2, 3)$ , substitute these values in the respective equations and solve for the unknown.

$$\begin{array}{ll} 2 = \frac{-1 + x_2}{2} & \text{and} \quad 3 = \frac{-5 + y_2}{2} \\ 4 = -1 + x_2 & 6 = -5 + y_2 \\ 5 = x_2 & 11 = y_2 \end{array}$$

The coordinates of the endpoint B are  $(5, 11)$ .

Another way is to use transformations. The student could realize that point C  $(2, 3)$  is a translation of point A  $(-1, -5)$  such that  $(x, y) \rightarrow (x + 3, y + 8)$  because  $-1 + 3 = 2$  and  $-5 + 8 = 3$ . Since C is the midpoint of AB, a point B must be the outcome of the same translation from C to B as it was from A to C, or  $(2 + 3, 3 + 8) = (5, 11)$ . Therefore, the coordinates of the endpoint B are  $(5, 11)$ .

### Sample Response: 1 point

Point A is located at  $(-1, -5)$ . The midpoint of line segment AB is point C  $(2, 3)$ .

What are the coordinates of point B?

(5.0, 11.0)

←	→	↶	↷	✕
1	2	3		
4	5	6		
7	8	9		
	0			
.	-	$\frac{\Box}{\Box}$		

#### Notes on Scoring

This response earns full credit (1 point) because it shows correctly identified coordinates of the point B.

The student provides the correct coordinates in an equivalent form.

### Sample Response: 0 points

Point A is located at  $(-1, -5)$ . The midpoint of line segment AB is point C  $(2, 3)$ .

What are the coordinates of point B?

(3, 8)

←	→	↶	↷	✖
1	2	3		
4	5	6		
7	8	9		
	0			
.	-	$\frac{\Box}{\Box}$		

#### Notes on Scoring

This response earns no credit (0 points) because it does not show the correct coordinates of the point B.

The student may incorrectly calculate the number of units point A is away from point C along the x-axis, or  $2 - (-1) = 3$ , and along the y-axis, or  $3 - (-5) = 8$  and use them as the coordinates of point B.

### Sample Response: 0 points

Point A is located at  $(-1, -5)$ . The midpoint of line segment AB is point C  $(2, 3)$ .

What are the coordinates of point B?

$(\frac{1}{2}, -1)$

←	→	↶	↷	✖
1	2	3		
4	5	6		
7	8	9		
	0			
.	-	$\frac{\Box}{\Box}$		

#### Notes on Scoring

This response earns no credit (0 points) because it does not show the correct coordinates of the point B.

The student may misuse the midpoint  $(2, 3)$  for the endpoint of a line segment and find the midpoint of AC, as  $\frac{-1+2}{2} = \frac{1}{2}$  and  $\frac{-5+3}{2} = -1$ . The student incorrectly uses  $(\frac{1}{2}, -1)$  as coordinates for point B.

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**Question 12**

**Question and Scoring Guidelines**

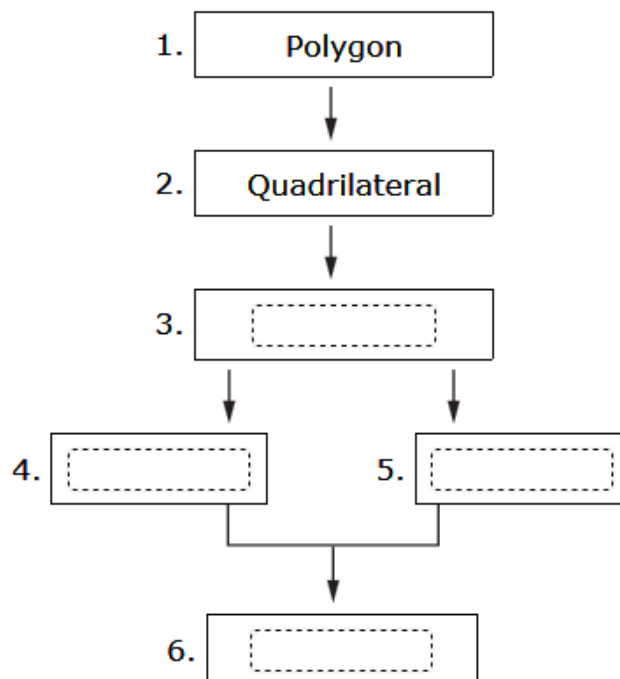


## Question 12

A partially completed chart shows the hierarchy of a set of polygons.

Move a term to each blank box to complete the chart.

Kite   Parallelogram   Rectangle   Rhombus   Square



**Points Possible:** 1

**Content Cluster:** Classify and analyze geometric figures.

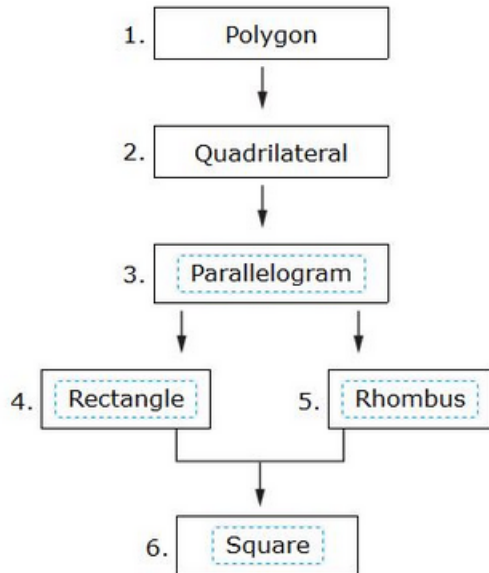
**Content Standard:** Classify two-dimensional figures in a hierarchy based on properties. (G.CO.14)

**Depth of Knowledge:** Level 2

- a. Classify plane and three-dimensional figures
- b. Interpret information from a simple graph

# Scoring Guidelines

## Exemplar Response



## Other Correct Responses

- Boxes 4 and 5 can be switched.

For the item, a full-credit response includes

- the correctly completed table (1 point).

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**Question 12**

**Sample Responses**

**Sample Response: 1 point**

A partially completed chart shows the hierarchy of a set of polygons.

Move a term to each blank box to complete the chart.

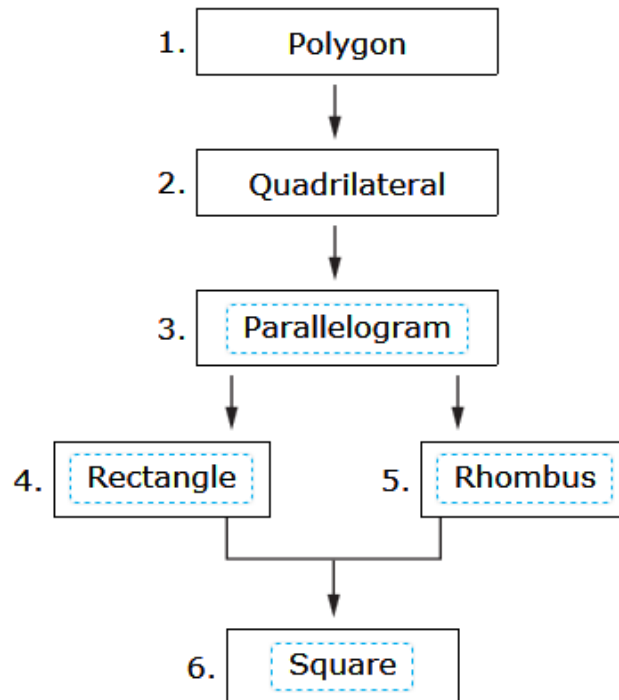
Kite

Parallelogram

Rectangle

Rhombus

Square



### Notes on Scoring

This response earns full credit (1 point) because it shows the correct classification of two-dimensional figures in the hierarchy based on their definitions and properties.

The item asks students to arrange major types of special polygons in such a way that under their formal definitions and properties, any statement about the type of quadrilateral would also be true for all types of quadrilaterals below it.

A kite can be defined as a quadrilateral with two distinct pairs of consecutive congruent sides. If students choose *Kite* as a type of quadrilateral to place in slot 3, then, based on congruence of sides, they can only place *Rhombus* and *Square* in slot 4 or 6, which leaves one empty slot, 5, for either *Parallelogram* or *Rectangle*. However, neither *Parallelogram* nor *Rectangle* can be placed in a slot beneath a kite because not every property of a kite will work for a rectangle or a parallelogram. So, *Kite* cannot be in slot 3. *Note: There is more than one definition of a kite, but the hierarchy represented by this diagram would be incorrect using either definition.*

A parallelogram is a quadrilateral with two pairs of opposite parallel and congruent sides. It implies that a parallelogram is not a kite (Note: A kite is only a parallelogram in the cases of a rhombus depending on the definition of a kite that is used). Since there is no direct connection between a kite and a parallelogram, *Kite* cannot be placed above or below *Parallelogram* in the given diagram and should be excluded from the hierarchy.

(continued on next page)

### Notes on Scoring

Therefore, no quadrilateral except for *Parallelogram* should be placed in slot 3. All properties of a parallelogram are true for all types of quadrilaterals beneath—*Rhombus*, *Rectangle* and *Square*.

The placement of a rectangle, a rhombus and a square is based on their definitions and their unique and common properties.

- A rectangle is a quadrilateral (or/and a parallelogram) with four right angles.
- A rhombus is a quadrilateral (or/and a parallelogram) with four congruent sides.
- A square is a quadrilateral (or/and a parallelogram) with four right angles and four congruent sides.

Since every rhombus and rectangle is a parallelogram, *Rectangle* can go in slot 4 and *Rhombus* can go in slot 5 (*Rectangle* and *Rhombus* can switch slots). Every square is both a rhombus and a rectangle because every square has four equal sides and has four right angles. Hence, *Square* goes in slot 6.

**Sample Response: 1 point**

A partially completed chart shows the hierarchy of a set of polygons.

Move a term to each blank box to complete the chart.

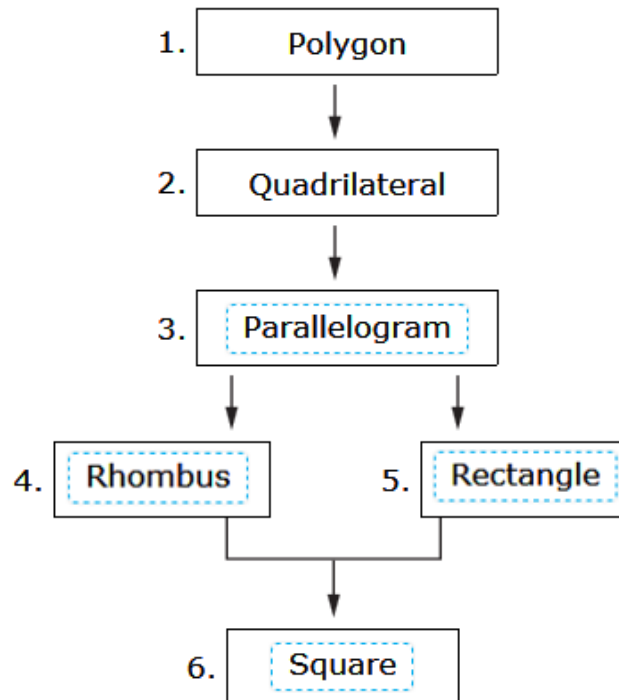
Kite

Parallelogram

Rectangle

Rhombus

Square



**Notes on Scoring**

This response earns full credit (1 point) because it shows the correct classification of two-dimensional figures in a hierarchy based on their definitions and properties.

Since every rhombus and rectangle is a parallelogram, *Rhombus* can go in slot 4 and *Rectangle* can go in slot 5 (*Rhombus* and *Rectangle* can switch slots). Every square is both a rhombus and a rectangle because every square has four equal sides and has four right angles. Hence, *Square* goes in slot 6.



### Sample Response: 0 points

A partially completed chart shows the hierarchy of a set of polygons.

Move a term to each blank box to complete the chart.

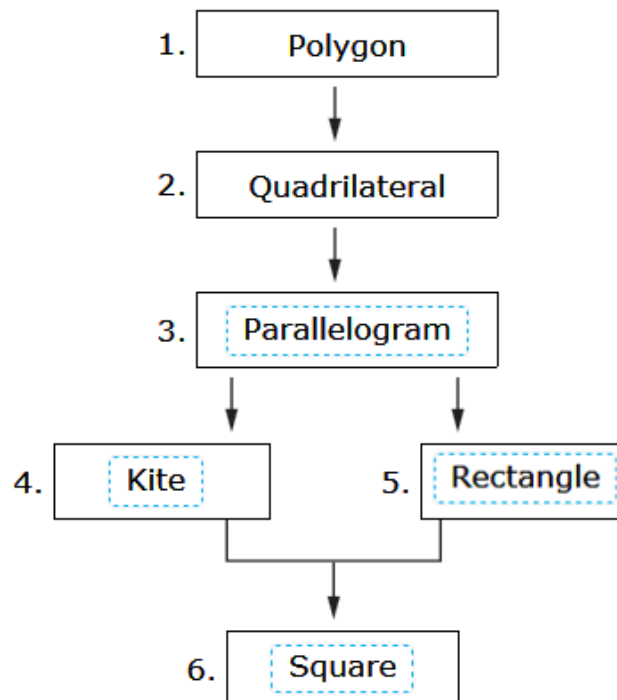
Kite

Parallelogram

Rectangle

Rhombus

Square



#### Notes on Scoring

This response earns no credit (0 points) because it shows an incorrect classification of two-dimensional figures in the hierarchy based on their definitions and properties.

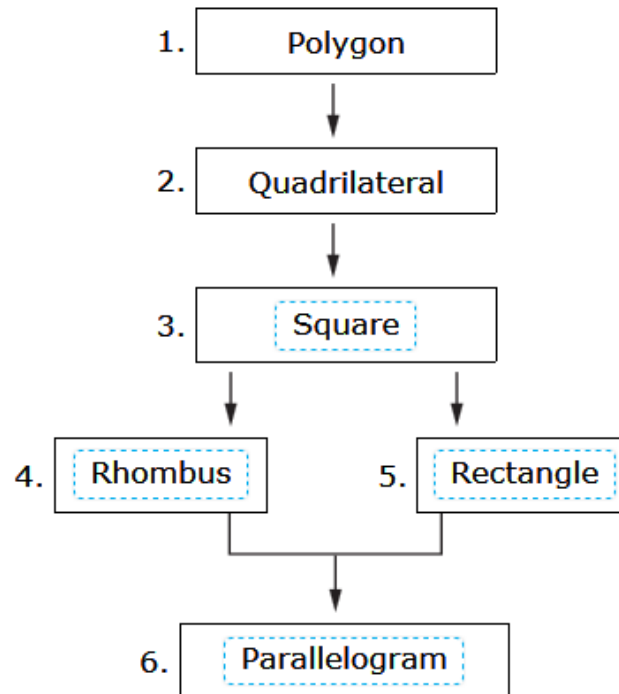
The student may think that kites are always parallelograms; however, per properties of parallelograms and kites, parallelograms do not always have perpendicular diagonals, but kites always have perpendicular diagonals.

### Sample Response: 0 points

A partially completed chart shows the hierarchy of a set of polygons.

Move a term to each blank box to complete the chart.

Kite   Parallelogram   Rectangle   Rhombus   Square



**Notes on Scoring**

This response earns no credit (0 points) because it shows an incorrect classification of two-dimensional figures in the hierarchy based on their definitions and properties.

The student may reverse the order of special polygons in the hierarchy in such way that under their formal definitions and properties, any statement about the type of quadrilateral would also be true for all types of quadrilaterals above it. Per this hierarchy, since all sides of a square are congruent, then all sides of a rectangle and a parallelogram are always congruent too, which is false.

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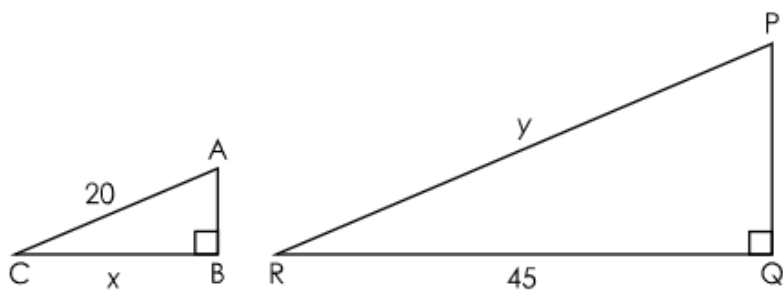
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**Question 16**

**Question and Scoring Guidelines**

## Question 16

A teacher asked Dwayne to find the values of  $x$  and  $y$  in the triangles shown.



The teacher provided the following information about the triangles:

- Triangle ABC is similar to triangle PQR.
- In triangle ABC,  $\cos(C) = 0.92$ .

Dwayne claims that the value of  $x$  can be determined but the information provided is not sufficient to find the value of  $y$ .

Which statement about Dwayne's claim is accurate?

- Ⓐ His claim is correct because  $\cos(C) = \frac{20}{x}$  and 0.92 can be substituted for  $\cos(C)$ , but the cosine of angle R is not given for triangle PQR.
- Ⓑ His claim is correct because  $\cos(C) = \frac{x}{20}$  and 0.92 can be substituted for  $\cos(C)$ , but the cosine of angle R is not given for triangle PQR.
- Ⓒ His claim is incorrect because  $\cos(C) = \frac{20}{x}$ , 0.92 can be substituted for  $\cos(C)$ , and since the triangles are similar, this ratio will be the same as  $\frac{y}{45}$ .
- Ⓓ His claim is incorrect because  $\cos(C) = \frac{x}{20}$ , 0.92 can be substituted for  $\cos(C)$ , and since the triangles are similar, this ratio will be the same as  $\frac{45}{y}$ .

**Points Possible:** 1

**Content Cluster:** Define trigonometric ratios and solve problems involving right triangles.

**Content Standard:** Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles. (*G.SRT.6*)

**Depth of Knowledge:** Level 3

- a. Interpret information from a complex graph
- c. Make and/or justify conjectures
- j. Provide mathematical justifications

## Scoring Guidelines

Rationale for Option A: This is incorrect. The student may incorrectly assume that the cosine of an angle is the ratio of the length of the hypotenuse to the length of the adjacent side, or  $\frac{20}{x}$ . The student may not recognize that in similar triangles, the ratios of corresponding sides are equal and that the given value of  $\cos(C)$  can be substituted for  $\cos(R)$  to find the missing lengths of triangle PQR.

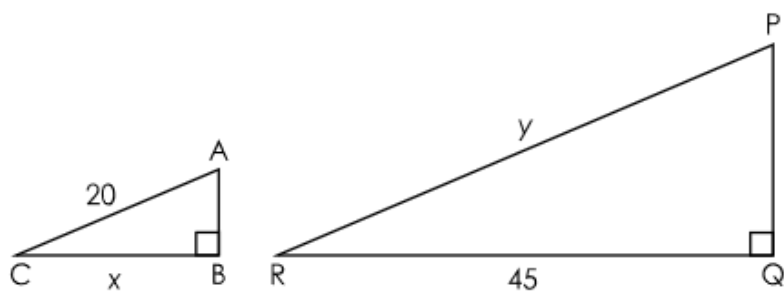
Rationale for Option B: This is incorrect. The student may not recognize that in similar triangles, the ratios of corresponding sides are equal and that the given value of  $\cos(C) = 0.92$  can be substituted for  $\cos(R)$  to find the missing lengths of triangle PQR.

Rationale for Option C: This is incorrect. The student may incorrectly assume that the cosine of an angle is the ratio of the length of the hypotenuse to the length of the adjacent side, or  $\frac{20}{x}$ , but recognize that ratios of corresponding sides are equal and that the given value of  $\cos(C)$  can be substituted for  $\cos(R)$  to find the missing lengths of triangle PQR.

Rationale for Option D: **Key** – The student correctly recognizes that since the cosine of an angle is the ratio of the length of the adjacent leg to the length of the hypotenuse, or  $\frac{x}{20}$ , the value of  $x$  can be determined from the equation  $\cos(C) = \frac{x}{20}$ , or  $0.92 = \frac{x}{20}$ . The student understands that in similar triangles, the measure of corresponding angles are equal, so that  $\cos(C) = \cos(R) = 0.92$ . Therefore, to find a value of  $y$ , use  $\cos(R) = \frac{45}{y}$ , or  $0.92 = \frac{45}{y}$  and then solve it for  $y$  as  $y = \frac{45}{0.92}$ .

### Sample Response: 1 point

A teacher asked Dwayne to find the values of  $x$  and  $y$  in the triangles shown.



The teacher provided the following information about the triangles:

- Triangle ABC is similar to triangle PQR.
- In triangle ABC,  $\cos(C) = 0.92$ .

Dwayne claims that the value of  $x$  can be determined but the information provided is not sufficient to find the value of  $y$ .

Which statement about Dwayne's claim is accurate?

- ☐ (A) His claim is correct because  $\cos(C) = \frac{20}{x}$  and 0.92 can be substituted for  $\cos(C)$ , but the cosine of angle R is not given for triangle PQR.
- ☐ (B) His claim is correct because  $\cos(C) = \frac{x}{20}$  and 0.92 can be substituted for  $\cos(C)$ , but the cosine of angle R is not given for triangle PQR.
- ☐ (C) His claim is incorrect because  $\cos(C) = \frac{20}{x}$ , 0.92 can be substituted for  $\cos(C)$ , and since the triangles are similar, this ratio will be the same as  $\frac{y}{45}$ .
- ☒ (D) His claim is incorrect because  $\cos(C) = \frac{x}{20}$ , 0.92 can be substituted for  $\cos(C)$ , and since the triangles are similar, this ratio will be the same as  $\frac{45}{y}$ .



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**Question 20**

**Question and Scoring Guidelines**

## Question 20

In triangle ABC,  $\angle A$  and  $\angle B$  are complementary, where  $\cos A = 0.5$ .

What is the measure, in degrees, of  $\angle B$ ?

*degrees*

←	→	↶	↷	✖
1	2	3		
4	5	6		
7	8	9		
	0			
.	-	$\frac{\square}{\square}$		

**Points Possible:** 1

**Content Cluster:** Define trigonometric ratios and solve problems involving right triangles.

**Content Standard:** Explain and use the relationship between the sine and cosine of complementary angles. (G.SRT.7)

**Depth of Knowledge:** Level 2

d. Solve a routine problem requiring multiple steps/decision points, or the application of multiple concepts

# Scoring Guidelines

## Exemplar Response

- 30 degrees

## Other Correct Responses

- any equivalent value

For the item, a full-credit response includes

- a correct angle measure (1 point).

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**Question 20**

**Sample Responses**

### Sample Response: 1 point

In triangle ABC,  $\angle A$  and  $\angle B$  are complementary, where  $\cos A = 0.5$ .

What is the measure, in degrees, of  $\angle B$ ?

*degrees*

← → ↶ ↷ ✕

1	2	3
4	5	6
7	8	9
	0	
.	-	$\frac{\square}{\square}$

#### Notes on Scoring

This response earns full credit (1 point) because it correctly identifies the measure of the complementary angle.

There is more than one way to approach this question.

One of them is to use a relationship between sine and cosine of complementary angles stating that if angles A and B are complementary, then  $\cos A = \sin B$ . In this situation, since  $\cos A = 0.5$ , then  $\sin B = 0.5$  as well.

By using Triangle Sum Theorem,  $m\angle A + m\angle B + m\angle C = 180$  and  $m\angle A + m\angle B = 90$  by a definition of complementary angles. Therefore,  $m\angle C = 90$ , and triangle ABC is a right triangle.

Based on ratios of special right triangles ( $30^\circ$ - $60^\circ$ - $90^\circ$ ), if sine of an angle B equals to 0.5, then the measure of angle B is 30 degrees.

### Sample Response: 1 point

In triangle ABC,  $\angle A$  and  $\angle B$  are complementary, where  $\cos A = 0.5$ .

What is the measure, in degrees, of  $\angle B$ ?

30.0

*degrees*

A digital calculator interface. At the top, there is a row of five circular navigation buttons: a left arrow, a right arrow, a double left arrow, a double right arrow, and a square with an 'X' (clear). Below these buttons is a numeric keypad with five rows of three buttons each. The buttons are labeled with digits 1 through 9, 0, a decimal point (.), a negative sign (-), and a fraction template icon (a square with a horizontal line and two smaller squares above and below). To the right of the keypad is a large, empty rectangular area for the display.

#### Notes on Scoring

This response earns full credit (1 point) because it correctly identifies the measure of the complementary angle.

The student provides the correct measure in an equivalent form.

### Sample Response: 0 points

In triangle ABC,  $\angle A$  and  $\angle B$  are complementary, where  $\cos A = 0.5$ .

What is the measure, in degrees, of  $\angle B$ ?

60

*degrees*

←

→

↶

↷

✖

1	2	3
4	5	6
7	8	9
	0	
.	-	$\frac{\square}{\square}$

#### Notes on Scoring

This response earns no credit (0 points) because it does not correctly identify the measure of the complementary angle.

The student may find the measure of angle A instead of the measure of an angle B. Based on ratios of special right triangles ( $30^\circ$ - $60^\circ$ - $90^\circ$ ), if cosine of angle A equals 0.5, then the measure of angle A is 60 degrees.

### Sample Response: 0 points

In triangle ABC,  $\angle A$  and  $\angle B$  are complementary, where  $\cos A = 0.5$ .

What is the measure, in degrees, of  $\angle B$ ?

89.5

*degrees*

←

→

↶

↷

✖

1	2	3
4	5	6
7	8	9
	0	
.	-	$\frac{\square}{\square}$

#### Notes on Scoring

This response earns no credit (0 points) because it does not correctly identify the measure of the complementary angle.

The student may confuse the cosine of A with the measure of A and based on the definition of complimentary angles think that if  $m\angle A = 0.5$ , then  $m\angle B = 90 - 0.5$  or 89.5 degrees.



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**Question 21**

**Question and Scoring Guidelines**

## Question 21

Trisha wants to create the perpendicular bisector of line segment AB.

She places her compass on point A and opens it with the width equal to the length of the line segment AB. She makes arcs above and below the line segment.

What could be Trisha's next step to create the perpendicular bisector of line segment AB?

- Ⓐ connect the two arcs using a straightedge
- Ⓑ connect each arc with point B using a straightedge
- Ⓒ place the compass on the approximate midpoint and draw intersecting arcs
- Ⓓ place the compass on point B and complete the same steps that she did for point A

**Points Possible:** 1

**Content Cluster:** Make geometric constructions.

**Content Standard:** Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line. (G.CO.12)

**Depth of Knowledge:** Level 1

g. Perform a specified or routine procedure (e.g., apply rules for rounding)

## Scoring Guidelines

Rationale for Option A: This is incorrect. The student may think about the final step of the construction when points of intersection of two sets of arcs should be connected.

Rationale for Option B: This is incorrect. The student may remember that the next step involves placing the compass on point B and connecting points where the arcs intersect but fail to remember that previously, arcs must be drawn from point B.

Rationale for Option C: This is incorrect. The student may remember that the next step involves drawing arcs again but forget that the arcs need to be from point B, not from the midpoint that has not been located yet.

Rationale for Option D: **Key** – The student correctly identifies that the next step is to place the compass on point B, open it with the width equal to the length of AB to make another pair of arcs above and below the line segment.

### Sample Response: 1 point

Trisha wants to create the perpendicular bisector of line segment AB.

She places her compass on point A and opens it with the width equal to the length of the line segment AB. She makes arcs above and below the line segment.

What could be Trisha's next step to create the perpendicular bisector of line segment AB?

- ☐ Ⓐ connect the two arcs using a straightedge
- ☐ Ⓑ connect each arc with point B using a straightedge
- ☐ Ⓒ place the compass on the approximate midpoint and draw intersecting arcs
- ☒ Ⓓ place the compass on point B and complete the same steps that she did for point A

**Geometry**  
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**Question 28**

**Question and Scoring Guidelines**

## Question 28

Bryan records the number of hours he sleeps each night for several days and whether it is raining in the morning when he wakes up. Bryan concludes that these two events are independent:

- Bryan sleeps 8 or more hours.
- It is raining in the morning.

Based on Bryan's conclusion, which statement must be true?

- (A) Bryan never sleeps 8 or more hours on days that it is not raining in the morning.
- (B) The probability that Bryan sleeps 8 or more hours is the same whether or not it is raining in the morning.
- (C) The probability that Bryan sleeps 8 or more hours is influenced by whether or not it is raining in the morning.
- (D) The probability that Bryan sleeps 8 or more hours is the same as the probability that it is raining in the morning.

**Points Possible:** 1

**Content Cluster:** Understand independence and conditional probability, and use them to interpret data.

**Content Standard:** Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer. (*S.CP.5*)

**Depth of Knowledge:** Level 2

e. Compare and/or contrast figures or statements

## Scoring Guidelines

Rationale for Option A: This is incorrect. The student may incorrectly think that if two events are independent, then the negation of the first event and the negation of the second event must happen together, but it only means that the probability of one event is not influenced by the other event occurring.

Rationale for Option B: **Key** – The student correctly realizes that if two events are independent, then the probability of one event happening is not influenced by the other event occurring. If the two events are independent, then the probability that Bryan sleeps 8 or more hours is not influenced by whether it rains in the morning.

Rationale for Option C: This is incorrect. The student may switch the meanings of independent and dependent events, since the probability of one event happening is influenced by the other event occurring only when events are dependent.

Rationale for Option D: This is incorrect. The student may think that if two events are independent, then they have equal probability of occurring, but it only means that their probabilities are not influenced by the other event occurring.

### Sample Response: 1 point

Bryan records the number of hours he sleeps each night for several days and whether it is raining in the morning when he wakes up. Bryan concludes that these two events are independent:

- Bryan sleeps 8 or more hours.
- It is raining in the morning.

Based on Bryan's conclusion, which statement must be true?

- ☐ Ⓐ Bryan never sleeps 8 or more hours on days that it is not raining in the morning.
- ☒ Ⓑ The probability that Bryan sleeps 8 or more hours is the same whether or not it is raining in the morning.
- ☐ Ⓒ The probability that Bryan sleeps 8 or more hours is influenced by whether or not it is raining in the morning.
- ☐ Ⓓ The probability that Bryan sleeps 8 or more hours is the same as the probability that it is raining in the morning.

**Geometry**  
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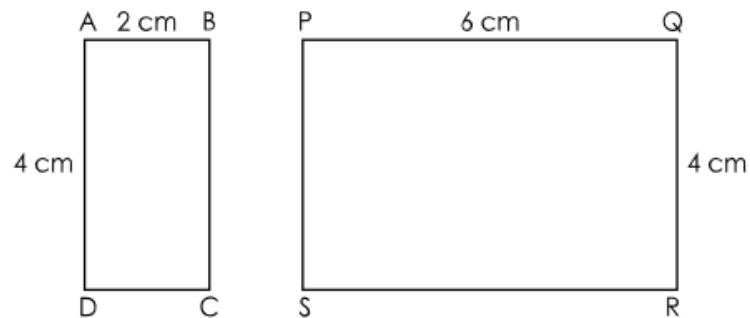
**Question 30**

**Question and Scoring Guidelines**



## Question 30

Consider the two rectangles shown.



Complete the sentence to determine whether the rectangles are similar.

Rectangle ABCD  similar to rectangle PQRS because , so  
rectangle ABCD  dilated to fit exactly over rectangle PQRS.

### Drop down choices

Rectangle ABCD  similar to rectangle PQRS because , so

is  
is not

all rectangles are similar  
all quadrilaterals are similar  
their corresponding sides are congruent  
their corresponding sides are not congruent  
their corresponding sides are proportional  
their corresponding sides are not proportional

rectangle ABCD  dilated to fit exactly over rectangle PQRS.

can be  
cannot be

**Points Possible:** 1

**Content Cluster:** Understand the relationships between lengths, area, and volumes.

**Content Standard:** Understand how and when changes to the measures of a figure (lengths or angles) result in similar and non-similar figures. (G.GMD.5)

**Depth of Knowledge:** Level 2

- b. Interpret information from a simple graph
- e. Compare and/or contrast figures or statements
- l. Select a procedure according to criteria and perform it

## Scoring Guidelines

### Exemplar Response

- Rectangle ABCD is not similar to rectangle PQRS because their corresponding sides are not proportional, so rectangle ABCD cannot be dilated to fit exactly over rectangle PQRS.

### Other Correct Responses

- N/A

For the item, a full-credit response includes

- the correctly completed sentence (1 point).

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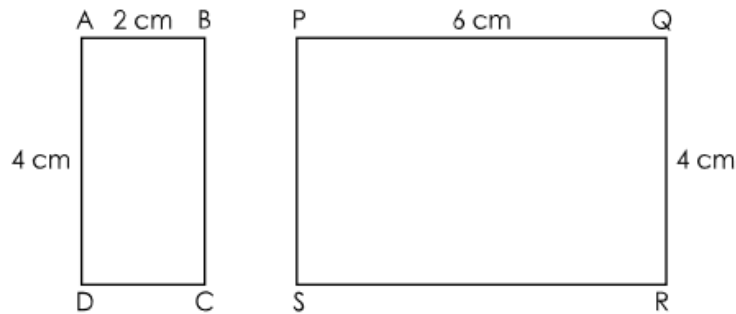
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**Question 30**

**Sample Responses**

## Sample Response: 1 point

Consider the two rectangles shown.



Complete the sentence to determine whether the rectangles are similar.

Rectangle ABCD  similar to rectangle PQRS because , so rectangle ABCD  dilated to fit exactly over rectangle PQRS.

### Notes on Scoring

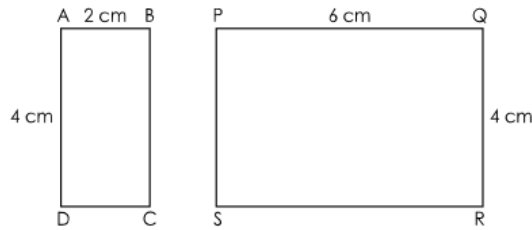
This response earns full credit (1 point) because it shows a correctly completed sentence to determine whether two rectangles are similar and why.

Two rectangles are similar if their corresponding side lengths are proportional. Also, two polygons are similar if one can be dilated by a constant scale factor to fit exactly over another one. A negation of these statements gives two new true statements such as two polygons are not similar if their corresponding side lengths are not proportional. Also, two polygons are not similar if one cannot be dilated by a constant scale factor to fit exactly over another one.

In this situation, use the criterion of proportionality to check similarity of rectangles by establishing and comparing ratios between corresponding side lengths of ABCD and PQRS, such as  $\frac{AB}{QR} = \frac{2}{4}$  and  $\frac{AD}{PQ} = \frac{4}{6}$  or  $\frac{AD}{QR} = \frac{4}{4}$  and  $\frac{AB}{PQ} = \frac{2}{6}$ . Since  $\frac{2}{4} \neq \frac{4}{6}$  and  $\frac{4}{4} \neq \frac{2}{6}$ , the side lengths are not proportional, so the rectangles are not similar. Altogether, it can be summarized in one sentence: Rectangle ABCD is not similar to rectangle PQRS because their corresponding sides are not proportional, so rectangle ABCD cannot be dilated to fit exactly over rectangle PQRS.

## Sample Response: 0 points

Consider the two rectangles shown.



Complete the sentence to determine whether the rectangles are similar.

Rectangle ABCD  similar to rectangle PQRS because , so rectangle ABCD  dilated to fit exactly over rectangle PQRS.

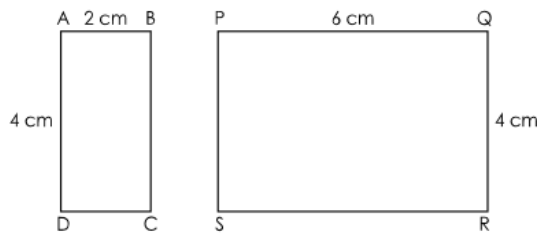
### Notes on Scoring

This response earns no credit (0 points) because it shows an incorrectly completed sentence to determine whether two rectangles are similar and why.

The student may incorrectly think that to be similar the rectangles must be congruent. Non-congruent rectangles are similar if their corresponding sides are proportional.

## Sample Response: 0 points

Consider the two rectangles shown.



Complete the sentence to determine whether the rectangles are similar.

Rectangle ABCD is  similar to rectangle PQRS because  their corresponding sides are congruent , so rectangle ABCD can be  dilated to fit exactly over rectangle PQRS.

### Notes on Scoring

This response earns no credit (0 points) because it shows an incorrectly completed sentence to determine whether two rectangles are similar and why.

The student may think that the presence of one pair of equal side lengths of 4 cm in the two rectangles is a sufficient condition for similarity. Instead, two perpendicular pairs of corresponding equal side lengths would mean that the two rectangles are congruent and thus similar.

**Geometry**  
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**Question 32**

**Question and Scoring Guidelines**

## Question 32

A university determined the number of students pursuing different degrees, by gender. Some of the results are shown.

	Undergraduate Degree	Master's Degree	Doctoral Degree	Total
Male			500	12,500
Female		2,500		12,500
Total	16,250	6,250	2,500	25,000

What is the probability that a female student chosen at random is pursuing an undergraduate degree?

- (A) 18%
- (B) 32%
- (C) 36%
- (D) 64%

**Points Possible:** 1

**Content Cluster:** Use the rules of probability to compute probabilities of compound events in a uniform probability model.

**Content Standard:** Find the conditional probability of  $A$  given  $B$  as the fraction of  $B$ 's outcomes that also belong to  $A$ , and interpret the answer in terms of the model. (*S.CP.6*)

**Depth of Knowledge:** Level 2

- i. Retrieve information from a table, graph, or figure and use it to solve a problem requiring multiple steps
- j. Translate between tables, graphs, words and symbolic notation



## Scoring Guidelines

Rationale for Option A: This is incorrect. The student may incorrectly add the number of female students pursuing a master's degree, 2500, to the number of female students pursuing a doctoral degree, 2000, ( $2500 + 500 = 3000$ ) to get 4500, divide it by the total number of students, 25000, and then identify the probability to be  $4500/25000 = 0.18$  or 18%.

Rationale for Option B: This is incorrect. The student may find the number of female undergraduate students to be 8000, ( $12500 - 2500 - 2000 = 8000$ ), divide that by the total number of students (25000) and then incorrectly identify the probability to be  $8000/25000 = 0.32$  or 32%.

Rationale for Option C: This is incorrect. The student may incorrectly add the number of female students pursuing a master's degree, 2500, to the number of female students pursuing a doctoral degree, 2000, ( $2500 + 500 = 3000$ ) to get 4500, divide it by the total number of female students, 12500, and then identify the probability to be  $4500/12500 = 0.36$  or 36%.

Rationale for Option D: **Key** – The student finds the number of female doctoral students (second row/third column) to be 2000 ( $2500 - 500 = 2000$ ); the number of female undergraduate students (second row/first column) to be 8000 ( $12500 - 2500 - 2000 = 8000$ ), and then determines the probability that a female student chosen at random is pursuing an undergraduate degree as  $8000/12500 = 0.64$  or 64%.

**Sample Response: 1 point**

A university determined the number of students pursuing different degrees, by gender. Some of the results are shown.

	<b>Undergraduate Degree</b>	<b>Master's Degree</b>	<b>Doctoral Degree</b>	<b>Total</b>
<b>Male</b>			500	12,500
<b>Female</b>		2,500		12,500
<b>Total</b>	16,250	6,250	2,500	25,000

What is the probability that a female student chosen at random is pursuing an undergraduate degree?

☐ (A) 18%

☐ (B) 32%

☐ (C) 36%

☒ 64%

**Geometry**  
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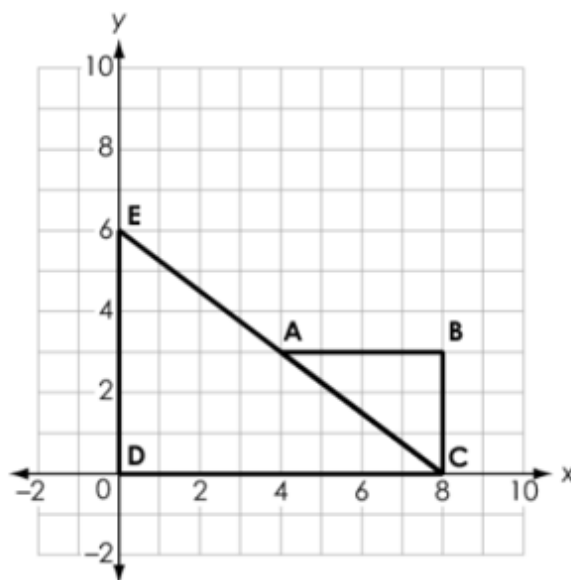
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**Question 33**

**Question and Scoring Guidelines**

## Question 33

Two triangles are shown on a coordinate grid.



Katie shows that the two triangles are similar by performing the following transformations:

- First, she rotates  $\triangle ABC$   $180^\circ$  about point A.
- Then, she dilates  $\triangle A'B'C'$  by a factor of  $k$  with a center of dilation at point A.
- Finally, she translates  $\triangle A''B''C''$   $p$  units to the right and  $q$  units down to map onto  $\triangle CDE$ .

What are the values of  $k$ ,  $p$ , and  $q$ ?

$k =$

$p =$

$q =$

<input type="button" value="←"/> <input type="button" value="→"/> <input type="button" value="↶"/> <input type="button" value="↷"/> <input type="button" value="✖"/>		
<input type="text" value="1"/>	<input type="text" value="2"/>	<input type="text" value="3"/>
<input type="text" value="4"/>	<input type="text" value="5"/>	<input type="text" value="6"/>
<input type="text" value="7"/>	<input type="text" value="8"/>	<input type="text" value="9"/>
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**Points Possible:** 2

**Content Cluster:** Understand similarity in terms of similarity transformations.

**Content Standard:** Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides. (*G.SRT.2*)

**Depth of Knowledge:** Level 2

- b. Interpret information from a simple graph
- e. Compare and/or contrast figures or statements
- i. Retrieve information from a table, graph, or figure and use it to solve a problem requiring multiple steps

## Scoring Guidelines

### Exemplar Response

- $k = 2$   
 $p = 4$   
 $q = 3$

### Other Correct Responses

- any equivalent value

For the item, a full-credit response includes

- a correct dilation factor (1 point)
- AND
- a correct pair of translations (1 point).

**Geometry**  
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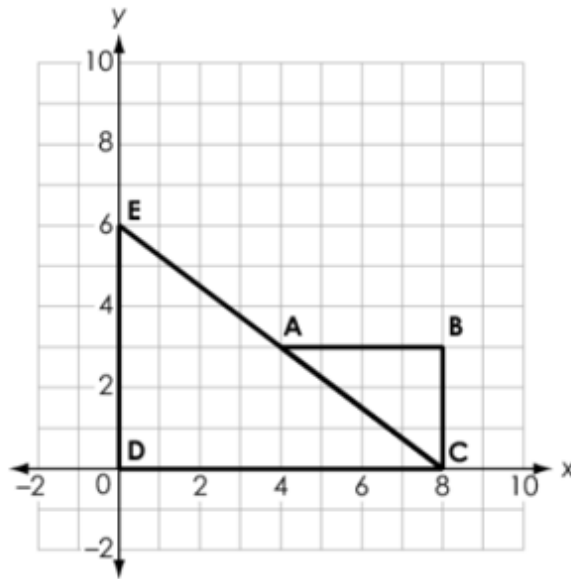
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**Question 33**

**Sample Responses**

## Sample Response: 2 points

Two triangles are shown on a coordinate grid.



Katie shows that the two triangles are similar by performing the following transformations:

- First, she rotates  $\triangle ABC$   $180^\circ$  about point A.
- Then, she dilates  $\triangle A'B'C'$  by a factor of  $k$  with a center of dilation at point A.
- Finally, she translates  $\triangle A''B''C''$   $p$  units to the right and  $q$  units down to map onto  $\triangle CDE$ .

What are the values of  $k$ ,  $p$ , and  $q$ ?

$$k = \boxed{2}$$

$$p = \boxed{4}$$

$$q = \boxed{3}$$

←	→	↶	↷	✖
1	2	3		
4	5	6		
7	8	9		
	0			
.	-	$\frac{\Box}{\Box}$		

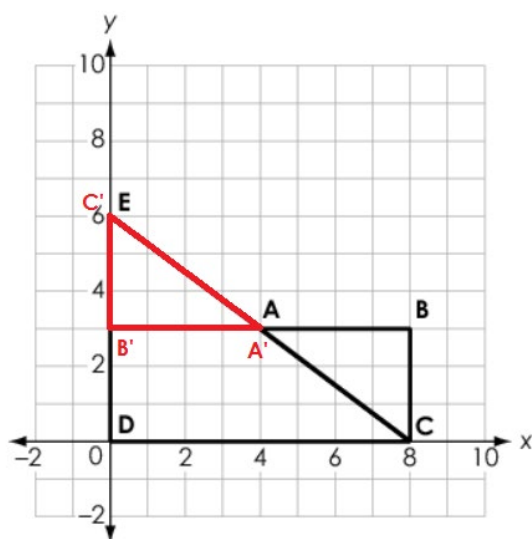
### Notes on Scoring

This response earns full credit (2 points) because it shows a correctly determined scale factor,  $k$ , correctly determined horizontal shift,  $p$ , and correctly determined vertical shift,  $q$ .

To show that two given triangles are similar they should map onto one another after the sequence of transformations.

In this situation, the first transformation to be performed is a rotation.

The rotation rotates the triangle ABC 180° about the point A and results in the triangle A'B'C' that is congruent to triangle ABC with the vertices at A'(4, 3), B'(0, 3) and C'(0, 6).



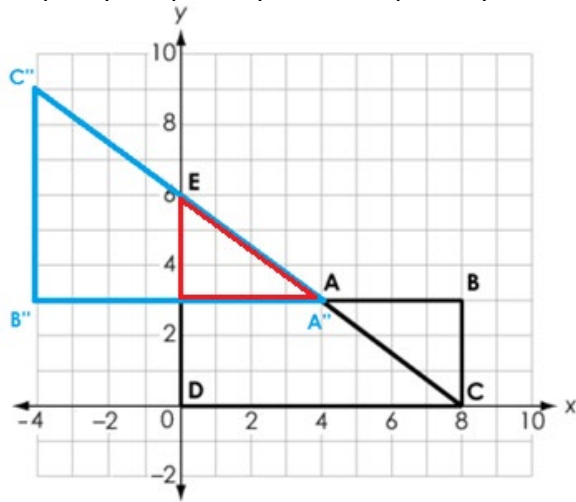
The two right triangles ABC and CDE are similar by the scale factor of 2 because corresponding side lengths are proportional since  $\frac{6}{3} = \frac{8}{4}$ , or  $\frac{2}{1}$ . Therefore, triangles A'B'C' and CDE are also similar with the same scale factor of 2.

(continued on next page)



### Notes on Scoring

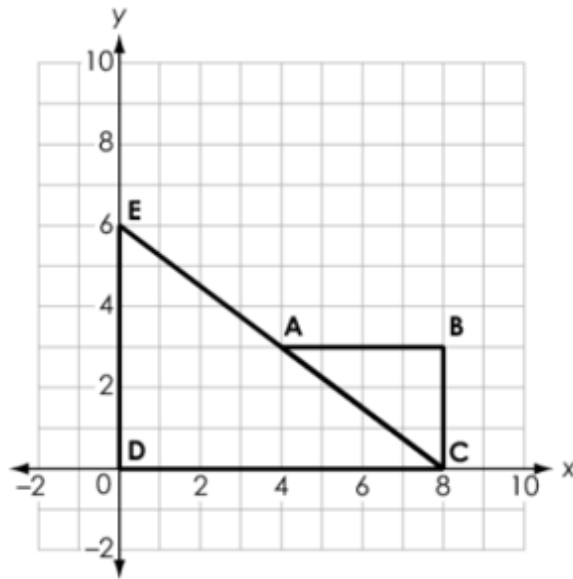
Therefore, triangle  $A'B'C'$  should undergo the second transformation, a dilation, with the center at  $A$  and the scale factor  $k = 2$ , so that the new triangle  $A''B''C''$  is congruent to the triangle  $CDE$ . Now, the triangle  $A''B''C''$  has vertices at  $A''(4, 3)$ ,  $B''(-4, 3)$  and  $C''(-4, 9)$ .



Finally, after translations of the triangle  $A''B''C''$  4 units right and 3 units down, the two triangles map onto one another.

### Sample Response: 2 points

Two triangles are shown on a coordinate grid.



Katie shows that the two triangles are similar by performing the following transformations:

- First, she rotates  $\triangle ABC$   $180^\circ$  about point A.
- Then, she dilates  $\triangle A'B'C'$  by a factor of  $k$  with a center of dilation at point A.
- Finally, she translates  $\triangle A''B''C''$   $p$  units to the right and  $q$  units down to map onto  $\triangle CDE$ .

What are the values of  $k$ ,  $p$ , and  $q$ ?

$$k = \frac{6}{3}$$

$$p = 4$$

$$q = 3$$

←	→	↶	↷	✖
1	2	3		
4	5	6		
7	8	9		
	0			
.	-	$\frac{\Box}{\Box}$		

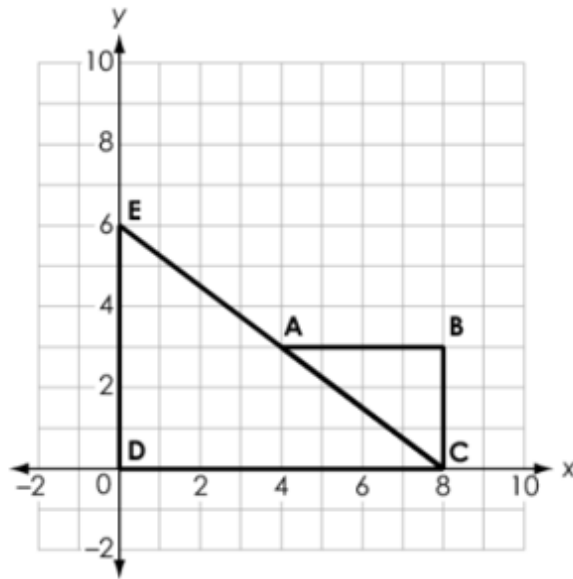
**Notes on Scoring**

This response earns full credit (2 points) because it shows a correctly determined scale factor,  $k$ , correctly determined horizontal shift,  $p$ , and correctly determined vertical shift,  $q$ .

The student provides the scale factor in an equivalent form.

### Sample Response: 1 point

Two triangles are shown on a coordinate grid.



Katie shows that the two triangles are similar by performing the following transformations:

- First, she rotates  $\triangle ABC$   $180^\circ$  about point A.
- Then, she dilates  $\triangle A'B'C'$  by a factor of  $k$  with a center of dilation at point A.
- Finally, she translates  $\triangle A''B''C''$   $p$  units to the right and  $q$  units down to map onto  $\triangle CDE$ .

What are the values of  $k$ ,  $p$ , and  $q$ ?

$$k = \boxed{3}$$

$$p = \boxed{4}$$

$$q = \boxed{3}$$

← → ↶ ↷ ✕		
1	2	3
4	5	6
7	8	9
	0	
.	-	$\frac{\Box}{\Box}$

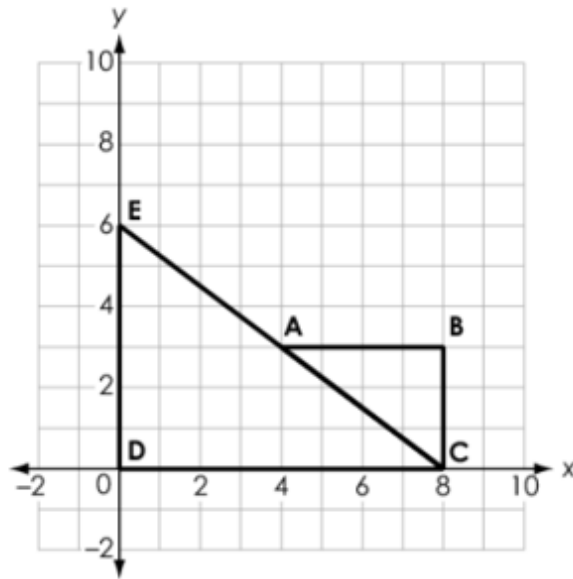
**Notes on Scoring**

This response earns a partial credit (1 point) because it shows a correctly determined horizontal shift,  $p$ , and correctly determined vertical shift,  $q$ , but an incorrectly determined scale factor,  $k$ .

The student may find the difference,  $6 - 3 = 3$ , between the lengths of the corresponding sides DE and BC and think that 3 is the scale factor instead of finding the quotient,  $\frac{6}{3} = 2$ .

Sample Response: 1 point

Two triangles are shown on a coordinate grid.



Katie shows that the two triangles are similar by performing the following transformations:

- First, she rotates  $\triangle ABC$   $180^\circ$  about point A.
- Then, she dilates  $\triangle A'B'C'$  by a factor of  $k$  with a center of dilation at point A.
- Finally, she translates  $\triangle A''B''C''$   $p$  units to the right and  $q$  units down to map onto  $\triangle CDE$ .

What are the values of  $k$ ,  $p$ , and  $q$ ?

$$k = \boxed{2}$$

$$p = \boxed{3}$$

$$q = \boxed{4}$$

←	→	↶	↷	✖
1	2	3		
4	5	6		
7	8	9		
	0			
.	-	$\frac{\Box}{\Box}$		

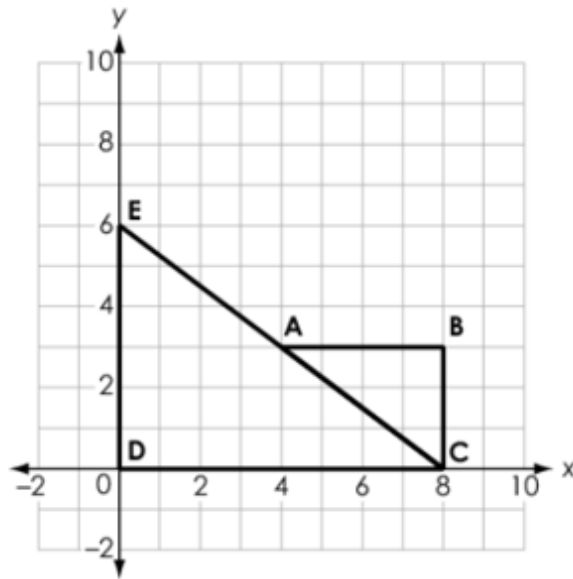
**Notes on Scoring**

This response earns partial credit (1 point) because it shows a correctly determined scale factor,  $k$ , but incorrectly determined values for the horizontal shift,  $p$ , and vertical shift,  $q$ .

The student may switch the values for the horizontal and vertical shifts.

## Sample Response: 0 points

Two triangles are shown on a coordinate grid.



Katie shows that the two triangles are similar by performing the following transformations:

- First, she rotates  $\triangle ABC$   $180^\circ$  about point A.
- Then, she dilates  $\triangle A'B'C'$  by a factor of  $k$  with a center of dilation at point A.
- Finally, she translates  $\triangle A''B''C''$   $p$  units to the right and  $q$  units down to map onto  $\triangle CDE$ .

What are the values of  $k$ ,  $p$ , and  $q$ ?

$$k = \boxed{3}$$

$$p = \boxed{3}$$

$$q = \boxed{4}$$

←	→	↶	↷	✕
1	2	3		
4	5	6		
7	8	9		
	0			
.	-	$\frac{\Box}{\Box}$		



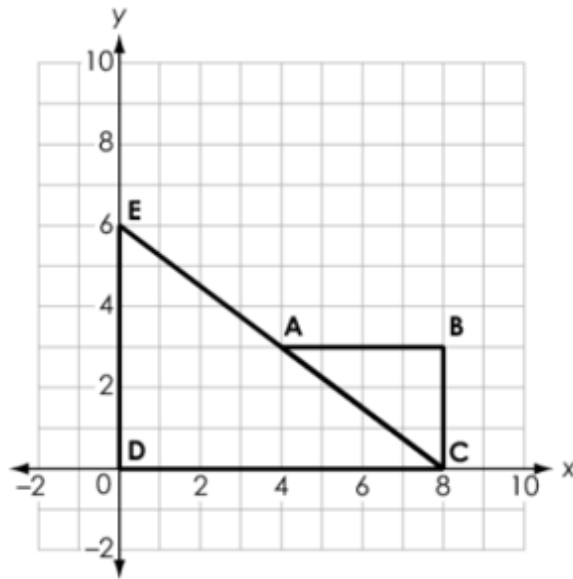
**Notes on Scoring**

This response earns no credit (0 points) because it shows an incorrectly determined scale factor,  $k$ , horizontal shift,  $p$ , and vertical shift,  $q$ .

The student may find the difference,  $6 - 3 = 3$ , between the lengths of the corresponding sides DE and BC and think that 3 is the scale factor. The student may then switch the values for the horizontal and vertical shifts.

### Sample Response: 0 points

Two triangles are shown on a coordinate grid.



Katie shows that the two triangles are similar by performing the following transformations:

- First, she rotates  $\triangle ABC$   $180^\circ$  about point A.
- Then, she dilates  $\triangle A'B'C'$  by a factor of  $k$  with a center of dilation at point A.
- Finally, she translates  $\triangle A''B''C''$   $p$  units to the right and  $q$  units down to map onto  $\triangle CDE$ .

What are the values of  $k$ ,  $p$ , and  $q$ ?

$$k = \frac{8}{3}$$

$$p = 3$$

$$q = 4$$

← → ↶ ↷ ✕		
1	2	3
4	5	6
7	8	9
	0	
.	-	$\frac{\Box}{\Box}$

**Notes on Scoring**

This response earns no credit (0 points) because it shows an incorrectly determined scale factor,  $k$ , horizontal shift,  $p$ , and vertical shift,  $q$ .

The student may find the scale factor using lengths of non-corresponding sides  $BC$  and  $DC$ . Then the student may switch the values for the horizontal and vertical shifts.

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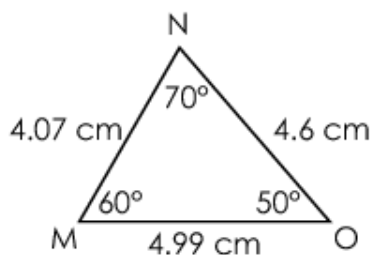
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**Question 34**

**Question and Scoring Guidelines**

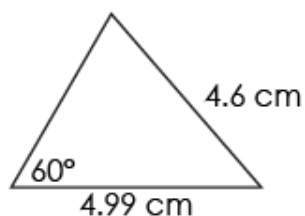
## Question 34

Triangle MNO is shown.

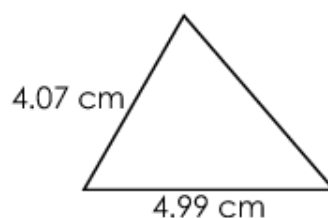


Which triangle can be shown to be congruent to triangle MNO with only the given information?

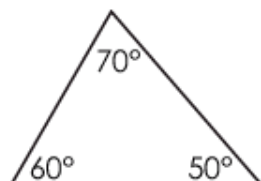
(A)



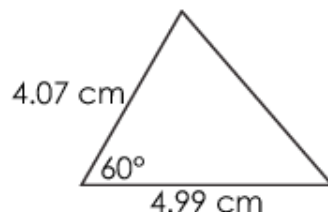
(C)



(B)



(D)



**Points Possible:** 1

**Content Cluster:** Understand congruence in terms of rigid motions.

**Content Standard:** Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions. (G.CO.8)

**Depth of Knowledge:** Level 1

- a. Recall, observe, or recognize a fact, definition, term, or property
- j. Retrieve information from a table or graph

## Scoring Guidelines

Rationale for Option A: This is incorrect. The student may not understand that the SSA criterion is not equivalent to the SAS criterion for congruence. To prove the congruence of triangles, the angles must be between two sides, or SAS.

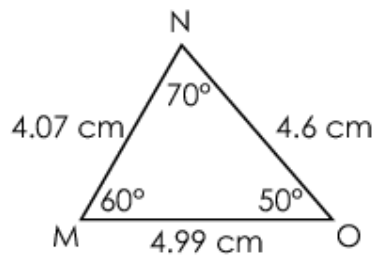
Rationale for Option B: This is incorrect. The student may think that since the presence of three pairs of congruent angles is a sufficient condition for similarity, then the presence of three pairs of congruent angles is a sufficient condition for congruence as well.

Rationale for Option C: This is incorrect. The student may think that if the presence of two pairs of congruent angles is a sufficient condition for similarity, then the presence of two pairs of congruent sides is a sufficient condition for congruence.

Rationale for Option D: **Key** – The student compares two triangles and notes that the presence of two pairs of congruent sides, 4.07 and 4.99 cm, and a pair of congruent included angles,  $60^\circ$ , satisfy the SAS criterion for congruence of two triangles.

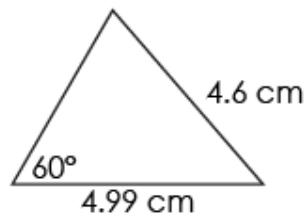
**Sample Response: 1 point**

Triangle MNO is shown.

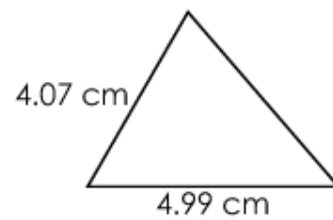


Which triangle can be shown to be congruent to triangle MNO with only the given information?

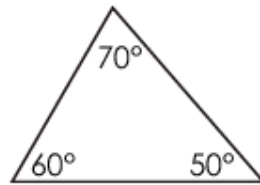
(A)



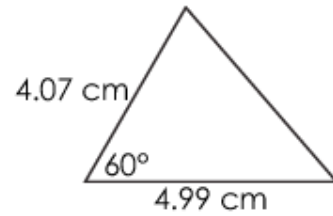
(C)



(B)



(D)



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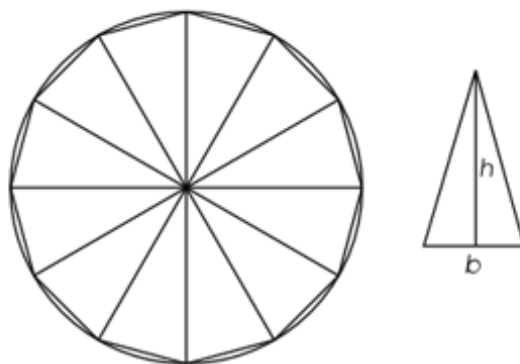
**Question 36**

**Question and Scoring Guidelines**



## Question 36

To estimate the area of the circle, Henry divides a circle of radius  $r$  into  $n$  triangles, as shown, and uses the expression  $\frac{h}{2} (b_1 + b_2 + \dots + b_n)$  to estimate the area of the circle. In the expression, variables  $b_1, b_2, \dots$  up to  $b_n$  represent the base lengths of each triangle and  $h$  represents the height of each triangle.



Henry claims that the more triangles the circle is divided into, the closer the estimated area will be to the actual area.

Which statement about Henry's claim is accurate?

- Ⓐ His claim is accurate because as  $n$  gets larger, the value of  $h$  gets closer to the value of  $r$  and the value of  $(b_1 + b_2 + \dots + b_n)$  approaches  $2\pi r$ .
- Ⓑ His claim is accurate because as  $n$  gets larger, the value of  $h$  gets closer to the value of  $2r$  and the value of  $(b_1 + b_2 + \dots + b_n)$  approaches  $\pi r$ .
- Ⓒ His claim is inaccurate because as  $n$  gets larger, the value of  $h$  gets closer to the value of  $r$  and the value of  $(b_1 + b_2 + \dots + b_n)$  deviates from  $2\pi r$ .
- Ⓓ His claim is inaccurate because as  $n$  gets larger, the value of  $h$  gets closer to the value of  $2r$  and the value of  $(b_1 + b_2 + \dots + b_n)$  deviates from  $\pi r$ .

**Points Possible:** 1

**Content Cluster:** Explain volume formulas and use them to solve problems.

**Content Standard:** Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. Use dissection arguments, Cavalieri's principle, and informal limit arguments. (G.GMD.1)

**Depth of Knowledge:** Level 3

- a. Interpret information from a complex graph
- e. Use concepts to solve non-routine problems
- m. Translate between a problem situation and symbolic notation that is not a direct translation

## Scoring Guidelines

Rationale for Option A: **Key** – The student correctly determines that the more triangles the circle is divided into, the closer the estimated area is to the actual area of a circle. The student correctly identifies that as  $n$  gets larger, the value of  $h$  approaches  $r$ , and the base of each triangle approaches the arc of the circle and the sum of the base lengths of the triangles approaches the circumference of the circle.

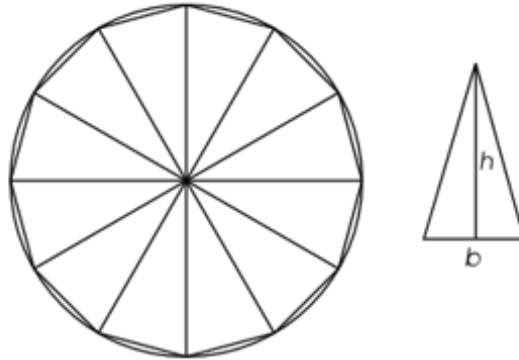
Rationale for Option B: This is incorrect. The student may correctly recognize that the more triangles the circle is divided into, the closer the estimated area is to the actual area of a circle but may incorrectly think that the value of  $h$  approaches  $2r$  as  $n$  gets larger. The student may have correctly realized that as  $n$  gets larger, the base of each triangle approaches the arc of the circle but incorrectly think that the sum of the base lengths of the triangles approaches a value equal to half the circumference of the circle, by confusing the height of the triangles for the diameter of the circle.

Rationale for Option C: This is incorrect. The student may incorrectly assume that the more triangles the circle is divided into, the further the estimated area is from the actual area of a circle. The student may correctly identify that the value of  $h$  approaches  $r$  as  $n$  gets larger but incorrectly determine that the sum of the base lengths of the triangles deviates from the value equal to the circumference of the circle instead of approaching that value.

Rationale for Option D: This is incorrect. The student may incorrectly assume that the more triangles the circle is divided into, the further the estimated area is from the actual area. The student may incorrectly think that the value of  $h$  approaches  $2r$ , by confusing the height of the triangles for the diameter of the circle, and incorrectly determine that as  $n$  gets larger, the sum of the base lengths of the triangles deviates from the value equal to the circumference of the circle instead of approaching that value.

### Sample Response: 1 point

To estimate the area of the circle, Henry divides a circle of radius  $r$  into  $n$  triangles, as shown, and uses the expression  $\frac{h}{2} (b_1 + b_2 + \dots + b_n)$  to estimate the area of the circle. In the expression, variables  $b_1, b_2, \dots, b_n$  represent the base lengths of each triangle and  $h$  represents the height of each triangle.



Henry claims that the more triangles the circle is divided into, the closer the estimated area will be to the actual area.

Which statement about Henry's claim is accurate?

- ☒ A His claim is accurate because as  $n$  gets larger, the value of  $h$  gets closer to the value of  $r$  and the value of  $(b_1 + b_2 + \dots + b_n)$  approaches  $2\pi r$ .
- ☐ B His claim is accurate because as  $n$  gets larger, the value of  $h$  gets closer to the value of  $2r$  and the value of  $(b_1 + b_2 + \dots + b_n)$  approaches  $\pi r$ .
- ☐ C His claim is inaccurate because as  $n$  gets larger, the value of  $h$  gets closer to the value of  $r$  and the value of  $(b_1 + b_2 + \dots + b_n)$  deviates from  $2\pi r$ .
- ☐ D His claim is inaccurate because as  $n$  gets larger, the value of  $h$  gets closer to the value of  $2r$  and the value of  $(b_1 + b_2 + \dots + b_n)$  deviates from  $\pi r$ .

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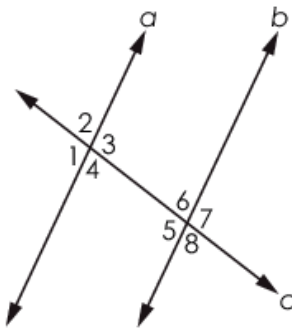
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**Question 40**

**Question and Scoring Guidelines**

## Question 40

Two parallel lines,  $a$  and  $b$ , are cut by a transversal  $c$  as shown.



Drag a statement or reason to each blank in the table to complete the proof that  $\angle 1 \cong \angle 7$ .

Statements	Reasons
1. $a \parallel b$	1. Given
2. <input type="text"/>	2. <input type="text"/>
3. <input type="text"/>	3. <input type="text"/>
4. $\angle 1 \cong \angle 7$	4. <input type="text"/>

$\angle 1 \cong \angle 3$

$\angle 1 \cong \angle 4$

$\angle 1 \cong \angle 5$

$\angle 3 \cong \angle 5$

$\angle 3 \cong \angle 7$

$\angle 4 \cong \angle 6$

$\angle 5 \cong \angle 7$

$\angle 6 \cong \angle 7$

Transitive property

Vertical angles are congruent.

Definition of supplementary angles.

Corresponding angles formed by parallel lines are congruent.

Alternate interior angles formed by parallel lines are congruent.

Alternate exterior angles formed by parallel lines are congruent.

**Points Possible:** 2

**Content Cluster:** Prove geometric theorems both formally and informally using a variety of methods.

**Content Standard:** Prove and apply theorems about lines and angles. Theorems include but are not restricted to the following: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints. (G.CO.9)

**Depth of Knowledge:** Level 3

- b. Explain thinking when more than one response is possible
- d. Use evidence to develop logical arguments for a concept
- j. Provide mathematical justifications

# Scoring Guidelines

## Exemplar Response

Statements	Reasons
1. $a \parallel b$	1. Given
2. $\angle 1 \cong \angle 3$	2. Vertical angles are congruent.
3. $\angle 3 \cong \angle 7$	3. Corresponding angles formed by parallel lines are congruent.
4. $\angle 1 \cong \angle 7$	4. Transitive property

## Other Correct Responses

Statements	Reasons
1. $a \parallel b$	1. Given
2. $\angle 1 \cong \angle 5$	2. Corresponding angles formed by parallel lines are congruent.
3. $\angle 5 \cong \angle 7$	3. Vertical angles are congruent.
4. $\angle 1 \cong \angle 7$	4. Transitive property

- Note: The order of lines 2 and 3 can be switched for full credit or partial credit.

For the item, a full-credit response includes

- three correct boxes (1 point)  
AND
- the remaining two correct boxes (1 point).



**Geometry**  
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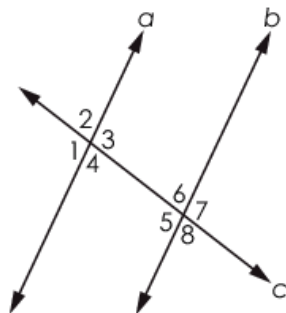
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**Question 40**

**Sample Responses**

Sample Response: 2 points

Two parallel lines,  $a$  and  $b$ , are cut by a transversal  $c$  as shown.



Drag a statement or reason to each blank in the table to complete the proof that  $\angle 1 \cong \angle 7$ .

Statements	Reasons
1. $a \parallel b$	1. Given
2. $\angle 1 \cong \angle 5$	2. Corresponding angles formed by parallel lines are congruent.
3. $\angle 5 \cong \angle 7$	3. Vertical angles are congruent.
4. $\angle 1 \cong \angle 7$	4. Transitive property

$\angle 1 \cong \angle 3$  $\angle 1 \cong \angle 4$  $\angle 1 \cong \angle 5$  $\angle 3 \cong \angle 5$

$\angle 3 \cong \angle 7$  $\angle 4 \cong \angle 6$  $\angle 5 \cong \angle 7$  $\angle 6 \cong \angle 7$

Transitive property

Vertical angles are congruent.

Definition of supplementary angles.

Corresponding angles formed by parallel lines are congruent.

Alternate interior angles formed by parallel lines are congruent.

Alternate exterior angles formed by parallel lines are congruent.

### Notes on Scoring

This response earns full credit (2 points) because it shows a correct placement of five answer options—two statements and three reasons—to complete the proof.

There is more than one correct way to complete this proof.

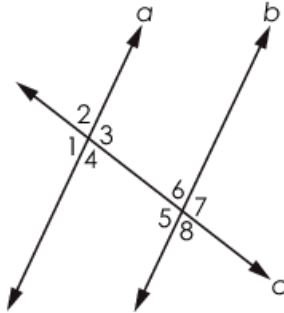
Since lines  $a$  and  $b$  are parallel and  $c$  is a transversal (given), the pairs of corresponding angles are congruent. Therefore, statement #2 is  $\angle 1 \cong \angle 5$  and reason #2 is Corresponding angles formed by parallel lines are congruent.

Lines  $a$ ,  $b$ , and  $c$  form four pairs of vertical angles. By the Vertical Angles Theorem, vertical angles are congruent. Therefore, statement #3 is  $\angle 5 \cong \angle 7$  and reason #3 is Vertical angles are congruent.

The use of the Transitive property (reason #4) completes the proof because statement #4 shows  $\angle 1 \cong \angle 7$  which follows from the two previous statements.

## Sample Response: 2 points

Two parallel lines,  $a$  and  $b$ , are cut by a transversal  $c$  as shown.



Drag a statement or reason to each blank in the table to complete the proof that  $\angle 1 \cong \angle 7$ .

Statements	Reasons
1. $a \parallel b$	1. Given
2. $\angle 1 \cong \angle 3$	2. Vertical angles are congruent.
3. $\angle 3 \cong \angle 7$	3. Corresponding angles formed by parallel lines are congruent.
4. $\angle 1 \cong \angle 7$	4. Transitive property

$\angle 1 \cong \angle 3$

$\angle 1 \cong \angle 4$

$\angle 1 \cong \angle 5$

$\angle 3 \cong \angle 5$

$\angle 3 \cong \angle 7$

$\angle 4 \cong \angle 6$

$\angle 5 \cong \angle 7$

$\angle 6 \cong \angle 7$

Transitive property

Vertical angles are congruent.

Definition of supplementary angles.

Corresponding angles formed by parallel lines are congruent.

Alternate interior angles formed by parallel lines are congruent.

Alternate exterior angles formed by parallel lines are congruent.

### Notes on Scoring

This response earns full credit (2 points) because it shows a correct placement of five correct answer options—two statements and three reasons—to complete the proof.

There is more than one correct way to complete this proof.

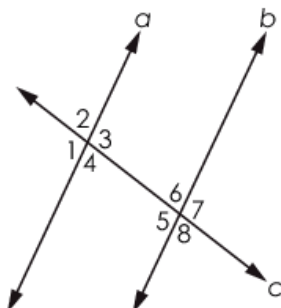
Lines  $a$ ,  $b$ , and  $c$  form four pairs of vertical angles. By the Vertical Angles Theorem, vertical angles are congruent. Therefore, statement #2 is  $\angle 1 \cong \angle 3$  and reason #2 is Vertical angles are congruent.

Because lines  $a$  and  $b$  are parallel, and  $c$  is a transversal (given), the pairs of corresponding angles are congruent. Therefore, statement #3 is  $\angle 3 \cong \angle 7$  and reason #3 is Corresponding angles formed by parallel lines are congruent.

The use of the Transitive property (reason #4) completes the proof because statement #4 shows  $\angle 1 \cong \angle 7$  which follows from the two previous statements.

## Sample Response: 1 point

Two parallel lines,  $a$  and  $b$ , are cut by a transversal  $c$  as shown.



Drag a statement or reason to each blank in the table to complete the proof that  $\angle 1 \cong \angle 7$ .

Statements	Reasons
1. $a \parallel b$	1. Given
2. $\angle 1 \cong \angle 3$	2. Corresponding angles formed by parallel lines are congruent.
3. $\angle 3 \cong \angle 7$	3. Vertical angles are congruent.
4. $\angle 1 \cong \angle 7$	4. Transitive property

$\angle 1 \cong \angle 3$

$\angle 1 \cong \angle 4$

$\angle 1 \cong \angle 5$

$\angle 3 \cong \angle 5$

$\angle 3 \cong \angle 7$

$\angle 4 \cong \angle 6$

$\angle 5 \cong \angle 7$

$\angle 6 \cong \angle 7$

Transitive property

Vertical angles are congruent.

Definition of supplementary angles.

Corresponding angles formed by parallel lines are congruent.

Alternate interior angles formed by parallel lines are congruent.

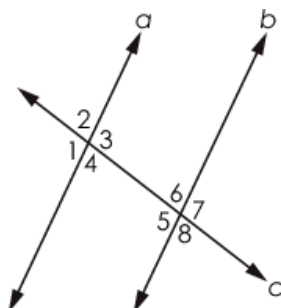
Alternate exterior angles formed by parallel lines are congruent.

### Notes on Scoring

This response earns partial credit (1 point) because the student incorrectly switches the placement of reasons #2 and #3.

## Sample Response: 1 point

Two parallel lines,  $a$  and  $b$ , are cut by a transversal  $c$  as shown.



Drag a statement or reason to each blank in the table to complete the proof that  $\angle 1 \cong \angle 7$ .

Statements	Reasons
1. $a \parallel b$	1. Given
2. $\angle 1 \cong \angle 3$	2. Vertical angles are congruent.
3. $\angle 3 \cong \angle 7$	3. Alternate interior angles formed by parallel lines are congruent.
4. $\angle 1 \cong \angle 7$	4. Alternate exterior angles formed by parallel lines are congruent.

$\angle 1 \cong \angle 3$

$\angle 1 \cong \angle 4$

$\angle 1 \cong \angle 5$

$\angle 3 \cong \angle 5$

$\angle 3 \cong \angle 7$

$\angle 4 \cong \angle 6$

$\angle 5 \cong \angle 7$

$\angle 6 \cong \angle 7$

Transitive property

Vertical angles are congruent.

Definition of supplementary angles.

Corresponding angles formed by parallel lines are congruent.

Alternate interior angles formed by parallel lines are congruent.

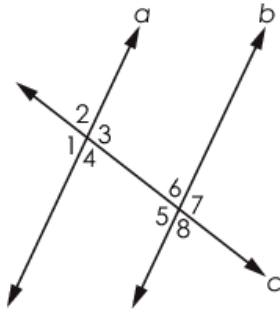
Alternate exterior angles formed by parallel lines are congruent.

### Notes on Scoring

This response earns partial credit (1 point) because it shows only 3 out of 5 correct placements, namely statements #2 and #3 and reason #2. Two remaining placements are incorrect. The student incorrectly switches the placement of statement #3 and #4.

## Sample Response: 0 points

Two parallel lines,  $a$  and  $b$ , are cut by a transversal  $c$  as shown.



Drag a statement or reason to each blank in the table to complete the proof that  $\angle 1 \cong \angle 7$ .

Statements	Reasons
1. $a \parallel b$	1. Given
2. $\angle 1 \cong \angle 4$	2. Definition of supplementary angles.
3. $\angle 4 \cong \angle 6$	3. Alternate interior angles formed by parallel lines are congruent.
4. $\angle 1 \cong \angle 7$	4. Definition of supplementary angles.

$\angle 1 \cong \angle 3$

$\angle 1 \cong \angle 4$

$\angle 1 \cong \angle 5$

$\angle 3 \cong \angle 5$

$\angle 3 \cong \angle 7$

$\angle 4 \cong \angle 6$

$\angle 5 \cong \angle 7$

$\angle 6 \cong \angle 7$

Transitive property

Vertical angles are congruent.

Definition of supplementary angles.

Corresponding angles formed by parallel lines are congruent.

Alternate interior angles formed by parallel lines are congruent.

Alternate exterior angles formed by parallel lines are congruent.

### Notes on Scoring

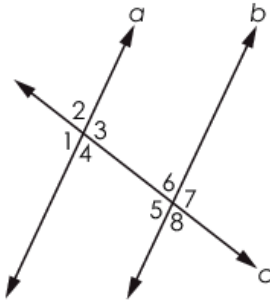
This response earns no credit (0 points) because it shows no correct placements.

The student bases their proof on an incorrect assumption that supplementary angles are congruent.



## Sample Response: 0 points

Two parallel lines,  $a$  and  $b$ , are cut by a transversal  $c$  as shown.



Drag a statement or reason to each blank in the table to complete the proof that  $\angle 1 \cong \angle 7$ .

Statements	Reasons
1. $a \parallel b$	1. Given
2. $\angle 6 \cong \angle 7$	2. Definition of supplementary angles.
3. $\angle 1 \cong \angle 4$	3. Definition of supplementary angles.
4. $\angle 1 \cong \angle 7$	4. Alternate exterior angles formed by parallel lines are congruent.

$\angle 1 \cong \angle 3$

$\angle 1 \cong \angle 4$

$\angle 1 \cong \angle 5$

$\angle 3 \cong \angle 5$

$\angle 3 \cong \angle 7$

$\angle 4 \cong \angle 6$

$\angle 5 \cong \angle 7$

$\angle 6 \cong \angle 7$

Transitive property

Vertical angles are congruent.

Definition of supplementary angles.

Corresponding angles formed by parallel lines are congruent.

Alternate interior angles formed by parallel lines are congruent.

Alternate exterior angles formed by parallel lines are congruent.

### Notes on Scoring

This response earns no credit (0 points) because it shows no correct placements.

The student bases the proof on an incorrect assumption that supplementary angles are congruent and on the use of irrelevant statements and reasons.

**Geometry**  
**Spring 2019 Item Release**

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**Question 41**

**Question and Scoring Guidelines**

## Question 41

A company wants to design a cylindrical object that has a height of 10 centimeters and a volume of at least 2,000 cubic centimeters, but not more than 2,500 cubic centimeters.

What is a possible radius, in centimeters, of the cylinder? Round your answer to the nearest hundredth.

*centimeters*

← → ↶ ↷ ✕

1	2	3
4	5	6
7	8	9
	0	
.	-	$\frac{\Box}{\Box}$

**Points Possible:** 1

**Content Cluster:** Apply geometric concepts in modeling situations.

**Content Standard:** Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios). (G.MG.3)

**Depth of Knowledge:** Level 2

- d. Solve a routine problem requiring multiple steps/decision points, or the application of multiple concepts
- k. Make direct translations between problem situations and symbolic notation
- l. Select a procedure according to criteria and perform it

# Scoring Guidelines

## Exemplar Response

- 7.98 centimeters

## Other Correct Responses

- any equivalent value from 7.977 to 8.923

For the item, a full-credit response includes

- a correct radius (1 point).

**Geometry**  
**Spring 2019 Item Release**

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**Question 41**

**Sample Responses**

**Sample Response: 1 point**

A company wants to design a cylindrical object that has a height of 10 centimeters and a volume of at least 2,000 cubic centimeters, but not more than 2,500 cubic centimeters.

What is a possible radius, in centimeters, of the cylinder? Round your answer to the nearest hundredth.

7.98

*centimeters*

<div><div>←</div><div>→</div><div>↶</div><div>↷</div><div>✖</div></div>		
1	2	3
4	5	6
7	8	9
	0	
.	-	$\frac{\Box}{\Box}$

### Notes on Scoring

This response earns full credit (1 point) because it shows a correctly identified possible value for radius of the cylindrical object that is consistent with the given dimensions and the range of the acceptable responses.

To design a cylinder with the fixed height of 10 cm and the volume that is not larger than 2,500 cubic centimeters but not smaller than 2,000 cubic centimeters, the company needs to determine the largest and the smallest radius of the base. The borderline (minimum and maximum) values for the radius can be found by using the formula for the volume of the cylinder,  $V = \pi r^2 h$ , where  $r$  is the radius of the base and  $h$  is the height of the cylinder. The equation for the volume of the cylinder with  $V = 2000$  is  $\pi r^2 \cdot 10 = 2000$  and the equation for the volume of the cylinder with  $V = 2500$  is  $\pi r^2 10 = 2500$ . Solve the two equations to obtain borderline values for the radius.

$$\pi r^2 10 = 2000$$

$$\pi r^2 = 200$$

$$r^2 = 200 \div \frac{22}{7} \text{ (using } \frac{22}{7} \text{ for } \pi)$$

$$r \approx \pm 7.977$$

$$\pi r^2 10 = 2500$$

$$\pi r^2 = 250$$

$$r^2 \approx \frac{250}{3.14} \text{ (using 3.14 for } \pi)$$

$$r \approx \pm 8.923$$

Since the radius of the cylinder can only be a positive number, only positive square roots must be considered. Depending on a value used for  $\pi$ , any real number between 7.977 and 8.923 is acceptable for the radius of the cylinder to keep the volume not larger than 2,500 cubic centimeters and not smaller than 2,000 cubic centimeters.

### Sample Response: 1 point

A company wants to design a cylindrical object that has a height of 10 centimeters and a volume of at least 2,000 cubic centimeters, but not more than 2,500 cubic centimeters.

What is a possible radius, in centimeters, of the cylinder? Round your answer to the nearest hundredth.

*centimeters*

←

→

↶

↷

✖

1	2	3
4	5	6
7	8	9
	0	
.	-	$\frac{\Box}{\Box}$

#### Notes on Scoring

This response earns full credit (1 point) because it shows a correctly identified value for the radius of the cylindrical object.

The student identifies a value that is consistent with the given dimensions and the range of the acceptable responses.



### Sample Response: 0 points

A company wants to design a cylindrical object that has a height of 10 centimeters and a volume of at least 2,000 cubic centimeters, but not more than 2,500 cubic centimeters.

What is a possible radius, in centimeters, of the cylinder? Round your answer to the nearest hundredth.

*centimeters*

←

→

↶

↷

✖

1	2	3
4	5	6
7	8	9
	0	
.	-	$\frac{\Box}{\Box}$

#### Notes on Scoring

This response earns no credit (0 points) because it shows a value for the radius of the cylindrical object that is inconsistent with the given dimensions and the range of the acceptable responses.

The student may create the correct equations for the borderline values of the radius using the volume of a cylinder formula but forget to apply a square root to both sides of the equations and round the answer down.

### Sample Response: 0 points

A company wants to design a cylindrical object that has a height of 10 centimeters and a volume of at least 2,000 cubic centimeters, but not more than 2,500 cubic centimeters.

What is a possible radius, in centimeters, of the cylinder? Round your answer to the nearest hundredth.

*centimeters*

←

→

↶

↷

✖

1	2	3
4	5	6
7	8	9
	0	
.	-	$\frac{\Box}{\Box}$

#### Notes on Scoring

This response earns no credit (0 points) because it shows a value for the radius of the cylindrical object that is inconsistent with the given dimensions and the range of the acceptable responses.

The student may create the equations for the borderline values of the radius using the formula for the volume of a cone instead of a cylinder and then accurately solve them for  $r$ :

$$\frac{1}{3}\pi r^2 10 = 2000$$

$$\pi r^2 = 600$$

$$r^2 \approx 190.9859$$

$$r \approx \pm 13.8198$$

$$\frac{1}{3}\pi r^2 10 = 2500$$

$$\pi r^2 = 750$$

$$r^2 \approx 238.7324$$

$$r \approx \pm 15.45097$$

**Geometry**  
**Spring 2019 Item Release**

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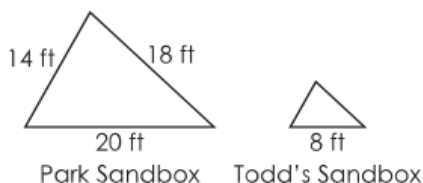
**Question 49**

**Question and Scoring Guidelines**

## Question 49

A park has a triangular sandbox. Todd wants to create a smaller sandbox at his backyard having the same angles as the park sandbox.

Drawings of both sandboxes are shown.



What is the perimeter, in feet (ft), of Todd's sandbox?

*ft*

←	→	↶	↷	✖
1	2	3		
4	5	6		
7	8	9		
	0			
.	-	$\frac{\Box}{\Box}$		

**Points Possible:** 1

**Content Cluster:** Understand the relationships between lengths, area, and volumes.

**Content Standard:** When figures are similar, understand and apply the fact that when a figure is scaled by a factor of  $k$ , the effect on lengths, areas, and volumes is that they are multiplied by  $k$ ,  $k^2$ , and  $k^3$ , respectively. (G.GMD.6)

**Depth of Knowledge:** Level 2

- b. Interpret information from a simple graph
- e. Compare and/or contrast figures or statements
- l. Select a procedure according to criteria and perform it

# Scoring Guidelines

## Exemplar Response

- 20.8 ft.

## Other Correct Responses

- any equivalent value

For the item, a full-credit response includes

- the correct perimeter (1 point).

**Geometry**  
**Spring 2019 Item Release**

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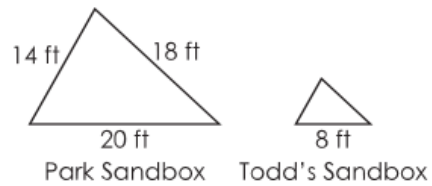
**Question 49**

**Sample Responses**

### Sample Response: 1 point

A park has a triangular sandbox. Todd wants to create a smaller sandbox at his backyard having the same angles as the park sandbox.

Drawings of both sandboxes are shown.



What is the perimeter, in feet (ft), of Todd's sandbox?

20.8 ft

←	→	↶	↷	✖
1	2	3		
4	5	6		
7	8	9		
	0			
.	-	$\frac{\Box}{\Box}$		

**Notes on Scoring**

This response earns full credit (1 point) because it shows a correctly identified perimeter of Todd's sandbox.

By the criterion of similar figures, Todd's sandbox is similar to the park sandbox because the boxes are triangles with three pairs of corresponding congruent angles. Since corresponding sides in similar triangles are proportional, the scale factor is  $\frac{8}{20}$  or  $\frac{2}{5}$ . The scale factor can be used either to find the missing side lengths of Todd's sandbox and then to find its perimeter, or to find a perimeter of the park sandbox and then multiply it by a scale factor to find the perimeter of Todd's sandbox.

The missing side lengths of Todd's sandbox are  $14 \cdot \frac{2}{5} = 5.6$  ft and  $18 \cdot \frac{2}{5} = 7.2$  ft. The perimeter of Todd's sandbox is the sum of three side lengths, or  $8 + 5.6 + 7.2 = 20.8$  ft.

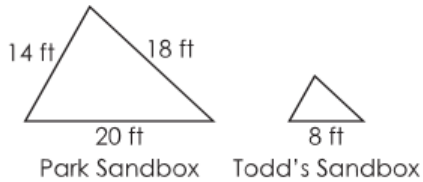
The perimeter of the park sandbox is the sum of three side lengths, or  $20 + 14 + 18 = 52$  ft. The perimeter of Todd's sandbox is  $52 \cdot \frac{2}{5} = 20.8$  ft.



## Sample Response: 1 point

A park has a triangular sandbox. Todd wants to create a smaller sandbox at his backyard having the same angles as the park sandbox.

Drawings of both sandboxes are shown.



What is the perimeter, in feet (ft), of Todd's sandbox?

$20\frac{4}{5}$  ft

←	→	↶	↷	✖
1	2	3		
4	5	6		
7	8	9		
	0			
.	-	$\frac{\Box}{\Box}$		

### Notes on Scoring

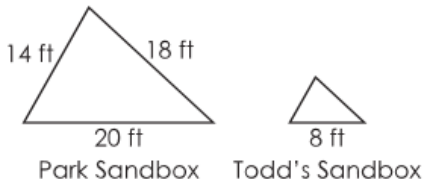
This response earns full credit (1 point) because it shows a correctly identified perimeter of Todd's sandbox.

The student provides the correct perimeter in an equivalent form.

## Sample Response: 0 points

A park has a triangular sandbox. Todd wants to create a smaller sandbox at his backyard having the same angles as the park sandbox.

Drawings of both sandboxes are shown.



What is the perimeter, in feet (ft), of Todd's sandbox?

16 ft

←	→	↶	↷	✖
1	2	3		
4	5	6		
7	8	9		
	0			
.	-	$\frac{\Box}{\Box}$		

### Notes on Scoring

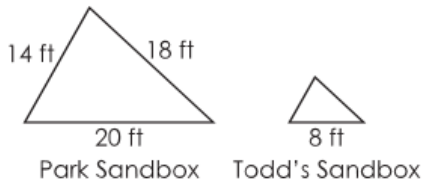
This response earns no credit (0 points) because it shows an incorrectly identified perimeter of Todd's sandbox.

The student may estimate the scale factor as 2, multiply the length of 8 ft for Todd's sandbox by 2 and then incorrectly use 16 ft as a perimeter of Todd's sandbox.

## Sample Response: 0 points

A park has a triangular sandbox. Todd wants to create a smaller sandbox at his backyard having the same angles as the park sandbox.

Drawings of both sandboxes are shown.



What is the perimeter, in feet (ft), of Todd's sandbox?

24 ft

←	→	↶	↷	✖
1	2	3		
4	5	6		
7	8	9		
	0			
.	-	$\frac{\Box}{\Box}$		

### Notes on Scoring

This response earns no credit (0 points) because it shows an incorrectly identified perimeter of Todd's sandbox.

The student may estimate the scale factor as 2 and use it to find missing side lengths of Todd's sandbox as  $\frac{14}{2} = 7$  ft and  $\frac{18}{2} = 9$  ft. Then, the student may find a perimeter by adding the three side lengths,  $8 + 7 + 9 = 24$  ft.