Ohio | Department of Education

# Ohio's State Tests

## ITEM RELEASE

## SPRING 2021

## GEOMETRY

# Table of Contents

# Geometry
# Spring 2021 Item Release
# Content Summary and Answer Key

| Question No.* | Item Type | Content Cluster | Content Standard | Depth of Knowledge | Answer Key | Points |
|---|---|---|---|---|---|---|
| 1 | Equation Item | Understand congruence in terms of rigid motions. | Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent. *(G.CO.7)* | Level 1 | --- | 1 point |
| 3 | Multiple Choice Item | Understand independ-ence and conditional probability, and use them to interpret data. | Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. *For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.★ (S.CP.4)* | Level 2 | B | 1 point |
| 4 | Equation Item | Explain volume formulas and use them to solve problems. | Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.★ *(G.GMD.3)* | Level 2 | --- | 1 point |

** The question number matches the item number in the Item Level Report in the Online Reporting System. The items are numbered sequentially in the practice site.*

# Geometry
# Spring 2021 Item Release
# Content Summary and Answer Key

| Question No.* | Item Type | Content Cluster | Content Standard | Depth of Knowledge | Answer Key | Points |
|---|---|---|---|---|---|---|
| 5 | Equation Item | Define trigonometric ratios, and solve problems involving right triangles. | Solve problems involving right triangles.<br>*a.* Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems if one of the two acute angles and a side length are given. ★ *(G.SRT.8)* | Level 3 | --- | 2 points |
| 11 | Equation Item | Understand independ-ence and conditional probability, and use them to interpret data. | Understand the conditional probability of A given B as $P(A \text{ and } B)/P(B)$, and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B.★ *(S.CP.3)* | Level 2 | --- | 1 point |
| 17 | Equation Item | Translate between the geometric description and the equation for a conic section. | Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation. *(G.GPE.1)* | Level 1 | --- | 1 point |
| 20 | Equation Item | Understand and apply theorems about circles. | Prove that all circles are similar using transformational arguments. *(G.C.1)* | Level 2 | --- | 1 point |
| 21 | Equation Item | Use the rules of probability to compute probabilities of compound events in a uniform probability model. | Apply the Addition Rule, P(A or B) = P(A) + P(B) - P(A and B), and interpret the answer in terms of the model.★ *(S.CP.7)* | Level 2 | --- | 1 point |

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# Geometry
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| Question No.* | Item Type | Content Cluster | Content Standard | Depth of Knowledge | Answer Key | Points |
|---|---|---|---|---|---|---|
| 22 | Multiple Choice Item | Prove and apply theorems both formally and informally involving similarity using a variety of methods. | Prove and apply theorems about triangles. *Theorems include but are not restricted to the following: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity. (G.SRT.4)* | Level 2 | D | 1 point |
| 23 | Inline Choice Item | Explain volume formulas and use them to solve problems. | Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. *Use dissection arguments, Cavalieri's principle, and informal limit arguments. (G.GMD.1)* | Level 2 | --- | 1 point |
| 25 | Grid Item | Experiment with transforma-tions in the plane. | Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another. *(G.CO.5)* | Level 2 | --- | 1 point |
| 28 | Equation Item | Understand similarity in terms of similarity transforma-tions. | Verify experimentally the properties of dilations[G] given by a center and a scale factor.<br>*b.* The dilation of a line segment is longer or shorter in the ratio given by the scale factor. *(G.SRT.1)* | Level 2 | --- | 1 point |

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# Geometry
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# Content Summary and Answer Key

| Question No.* | Item Type | Content Cluster | Content Standard | Depth of Knowledge | Answer Key | Points |
|---|---|---|---|---|---|---|
| 37 | Gap Match Item | Prove geometric theorems both formally and informally using a variety of methods. | Prove and apply theorems about lines and angles. *Theorems include but are not restricted to the following: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints. (G.CO.9)* | Level 3 | --- | 1 point |
| 40 | Equation Item | Prove geometric theorems both formally and informally using a variety of methods. | Prove and apply theorems about triangles. *Theorems include but are not restricted to the following: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point. (G.CO.10)* | Level 2 | --- | 1 point |
| 41 | Equation Item | Use the rules of probability to compute probabilities of compound events in a uniform probability model. | Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and interpret the answer in terms of the model.★ *(S.CP.6)* | Level 2 | --- | 1 point |

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|---|---|---|---|---|---|---|
| 42 | Inline Choice Item | Understand and apply theorems about circles. | Construct the inscribed and circumscribed circles of a triangle; prove and apply the property that opposite angles are supplementary for a quadrilateral inscribed in a circle. *(G.C.3)* | Level 3 | --- | 1 point |
| 43 | Multi-Select Item | Experiment with transforma-tions in the plane. | Identify the symmetries of a figure, which are the rotations and reflections that carry it onto itself. *(G.CO.3)* | Level 2 | B, C, D | 1 point |
| 45 | Equation Item | Use coordinates to prove simple geometric theorems algebraically and to verify specific geometric statements. | Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.★ *(G.GPE.7)* | Level 3 | --- | 1 point |
| 47 | Table Item | Experiment with transforma-tions in the plane. | Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch). *(G.CO.2)* | Level 2 | --- | 1 point |

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# Geometry
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# Content Summary and Answer Key

| Question No.* | Item Type | Content Cluster | Content Standard | Depth of Knowledge | Answer Key | Points |
|---|---|---|---|---|---|---|
| 48 | Multiple Choice Item | Understand similarity in terms of similarity transforma-tions. | Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar. *(G.SRT.3)* | Level 1 | C | 1 point |
| 49 | Multiple Choice Item | Understand congruence in terms of rigid motions. | Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions. *(G.CO.8)* | Level 1 | C | 1 point |

** The question number matches the item number in the Item Level Report in the Online Reporting System. The items are numbered sequentially in the practice site.*

# Depth of Knowledge (DOK)

DOK refers to the complexity of thinking required to complete a task in a given item. Items with a DOK 1 designation focus on the recall of information, such as definitions and terms, and simple procedures. Items with a DOK 2 designation require students to make decisions, solve routine problems, perform calculations, or recognize patterns. Items with a DOK 3 designation feature higher-order cognitive tasks. These DOK 3 tasks include but are not limited to: critiquing a statement and forming a conclusion; explaining, justifying, or proving a statement; or approaching abstract, complex, open-ended, and non-routine problems. Each grade's blueprint contains information about the number of points of opportunity students will encounter at each DOK level.

**Table 1: *Math Descriptors* – Applying Depth of Knowledge Levels for Mathematics (Webb, 2002) & NAEP 2002 Mathematics Levels of Complexity**
(M. Petit, Center for Assessment 2003, K. Hess, Center for Assessment, updated 2006)

| Level 1<br>Recall | Level 2<br>Skills/Concepts | Level 3<br>Strategic Thinking | Level 4*<br>Extended Thinking |
|---|---|---|---|
| a. Recall, observe, or recognize a fact, definition, term, or property<br>b. Apply/compute a well-known algorithm (e.g., sum, quotient)<br>c. Apply a formula<br>d. Determine the area or perimeter of rectangles or triangles given a drawing and labels<br>e. Identify a plane or three-dimensional figure<br>f. Measure<br>g. Perform a specified or routine procedure (e.g., apply rules for rounding)<br>h. Evaluate an expression<br>i. Solve a one-step word problem<br>j. Retrieve information from a table or graph | a. Classify plane and three-dimensional figures<br>b. Interpret information from a simple graph<br>c. Use models to represent mathematical concepts<br>d. Solve a routine problem requiring multiple steps/ decision points, or the application of multiple concepts<br>e. Compare and/or contrast figures or statements<br>f. Construct 2-dimensional patterns for 3-dimensional models, such as cylinders and cones<br>g. Provide justifications for steps in a solution process<br>h. Extend a pattern | a. Interpret information from a complex graph<br>b. Explain thinking when more than one response is possible<br>c. Make and/or justify conjectures<br>d. Use evidence to develop logical arguments for a concept<br>e. Use concepts to solve non-routine problems<br>f. Perform a procedure with multiple steps and multiple decision points<br>g. Generalize a pattern<br>h. Describe, compare, and contrast solution methods<br>i. Formulate a mathematical model for a complex situation<br>j. Provide mathematical justifications | a. Relate mathematical concepts to other content areas<br>b. Relate mathematical concepts to real-world applications in new situations<br>c. Apply a mathematical model to illuminate a problem, situation<br>d. Conduct a project that specifies a problem, identifies solution paths, solves the problem, and reports results<br>e. Design a mathematical model to inform and solve a practical or abstract situation<br>f. Develop generalizations of the results obtained and the strategies used and apply them to new problem situations |

**Table 1** *continued on next page.*

| Level 1<br>Recall | Level 2<br>Skills/Concepts | Level 3<br>Strategic Thinking | Level 4*<br>Extended Thinking |
|---|---|---|---|
| k. Recall, identify, or make conversions between and among representations or numbers (fractions, decimals, and percents), or within and between customary and metric measures<br>l. Locate numbers on a number line, or points on a coordinate grid<br>m. Solve linear equations<br>n. Represent math relationships in words, pictures, or symbols<br>o. Read, write, and compare decimals in scientific notation | i. Retrieve information from a table, graph, or figure and use it to solve a problem requiring multiple steps<br>j. Translate between tables, graphs, words and symbolic notation<br>k. Make direct translations between problem situations and symbolic notation<br>l. Select a procedure according to criteria and perform it<br>m. Specify and explain relationships between facts, terms, properties, or operations<br>n. Compare, classify, organize, estimate, or order data | k. Solve a multiple-step problem and provide support with a mathematical explanation that justifies the answer<br>l. Solve 2-step linear equations/ inequalities in one variable over the rational numbers, interpret solution(s) in the original context, and verify reasonableness of results<br>m. Translate between a problem situation and symbolic notation that is not a direct translation<br>n. Formulate an original problem, given a situation<br>o. Analyze the similarities and differences between procedures<br>p. Draw conclusion from observations or data, citing evidence | g. Apply one approach among many to solve problems<br>h. Apply understanding in a novel way, providing an argument/ justification for the application<br><br>***NOTE:*** *Level 4 involves such things as complex restructuring of data or establishing and evaluating criteria to solve problems.* |

***Note: Ohio's State Tests only assess and measure DOK Levels 1 – 3 in grades K – 12. Level 4 is included in this table for informational purposes only.***

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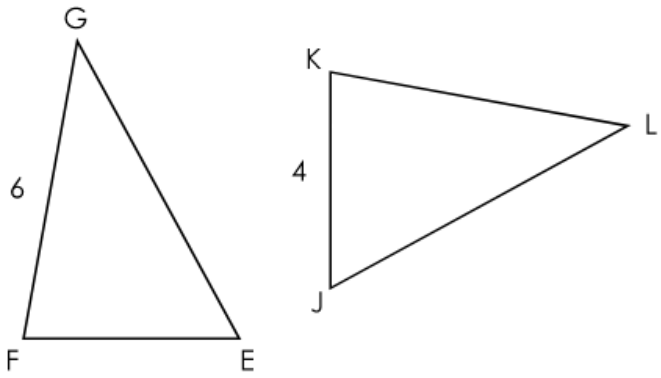
# Geometry
# Spring 2021 Item Release

# Question 1

# Question and Scoring Guidelines

# Question 1

Triangle EFG is congruent to triangle JKL.




What is KL, in units?

units

**Points Possible:** 1

**Content Cluster:** Understand congruence in terms of rigid motions.

**Content Standard:** Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent. *(G.CO.7)*

**Depth of Knowledge:** Level 1
a. Recall, observe, or recognize a fact, definition, term, or property

# Scoring Guidelines

Exemplar Response

- 6 units

Other Correct Responses

- any equivalent value

For this item, a full-credit response includes:

- the correct length (1 point).

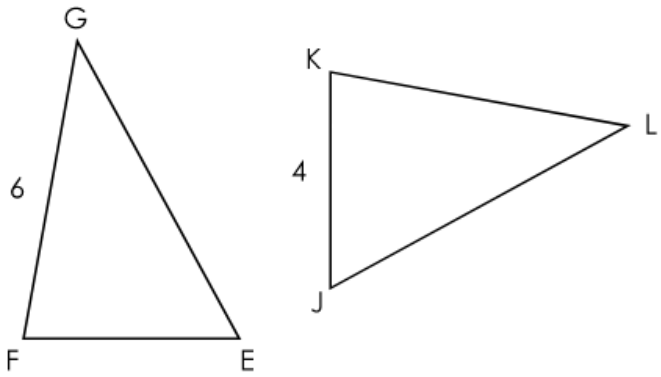
# Geometry
# Spring 2021 Item Release

# Question 1

# Sample Responses

**Sample Response: 1 point**

Triangle EFG is congruent to triangle JKL.




What is KL, in units?

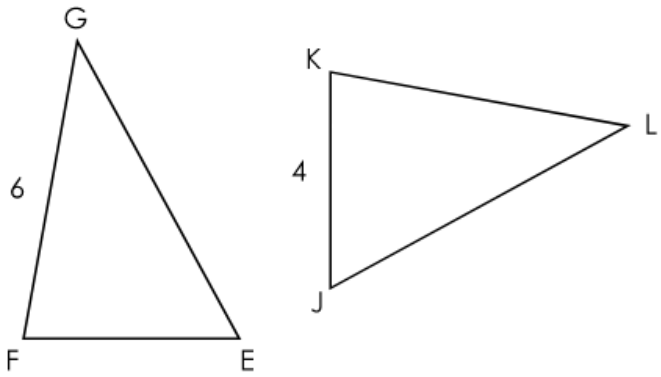
6 *units*

**Notes on Scoring**

This response earns full credit (1 point) because it shows a correct length for the side of the triangle. Since triangles EFG and JKL are congruent, one of them can undergo a sequence of transformations, a rotation and a translation, so that they coincide, which would make their corresponding sides and angles congruent. The student may mentally rotate the triangle JKL until it is the same orientation as triangle EFG and then translate it until vertices of triangles coincide. As a result of these transformations, point J is the image of point E, point K is the image of point F and point L is the image of point G. Therefore, side EF corresponds to side JK, side FG corresponds to side KL, and side EG corresponds to side JL. So, side KL equals 6 units because side FG is 6 units.

**Sample Response: 1 point**

Triangle EFG is congruent to triangle JKL.




What is KL, in units?

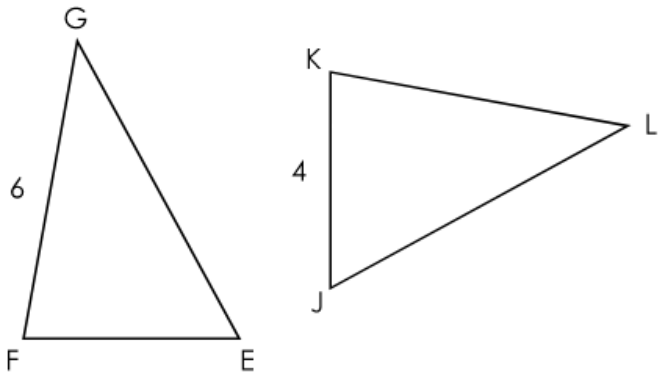
6.00 *units*

**Notes on Scoring**

This response earns full credit (1 point) because it shows a correct length for the side of the triangle. Since triangles EFG and JKL are congruent, one of them can undergo a sequence of transformations, a rotation and a translation, so that they coincide, which would make their corresponding sides and angles congruent. The student may mentally rotate the triangle JKL until it is the same orientation as triangle EFG and then translate it until vertices of triangles coincide. As a result of these transformations, point J is the image of point E, point K is the image of point F and point L is the image of point G. Therefore, side EF corresponds to side JK, side FG corresponds to side KL, and side EG corresponds to side JL. So, side KL equals 6 units because side FG is 6 units or 6.00 units.

## Sample Response: 0 points

Triangle EFG is congruent to triangle JKL.




What is KL, in units?

8 *units*

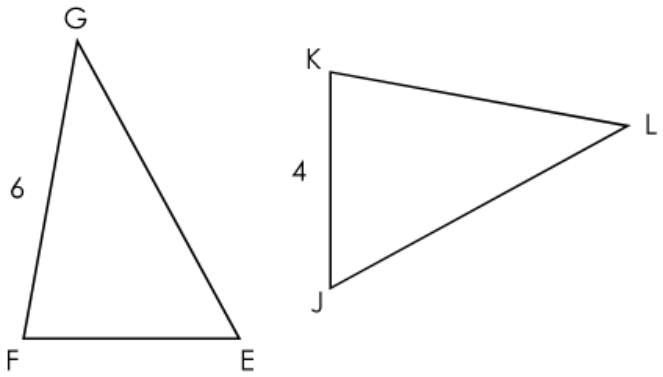
| | | |
|---|---|---|
| 1 | 2 | 3 |
| 4 | 5 | 6 |
| 7 | 8 | 9 |
| | 0 | |
| . | - | |

**Notes on Scoring**

This response earns no credit (0 points) because it shows an incorrect length for the side of the triangle. The student may think that since FG is 2 greater than JK, KL will be 2 greater than FG, so KL = 6 + 2 = 8.

## Sample Response: 0 points

Triangle EFG is congruent to triangle JKL.




What is KL, in units?

5 *units*

| | | |
|---|---|---|
| 1 | 2 | 3 |
| 4 | 5 | 6 |
| 7 | 8 | 9 |
| | 0 | |
| . | - | |

**Notes on Scoring**

This response earns no credit (0 points) because it shows an incorrect length for the side of the triangle. The student may incorrectly think that the length of the side KL must have a length that is between the two sides lengths shown on the diagram.

# Geometry
# Spring 2021 Item Release

# Question 3

# Question and Scoring Guidelines

# Question 3

The table shows the voting results of a mayoral election in a small town between candidates Smith and Jones. The small town has two polling places, one in the North District and one in the South District.

| | Smith | Jones | Total |
|---|---|---|---|
| **North District** | 21 | 39 | 60 |
| **South District** | 135 | 45 | 180 |
| **Total** | 156 | 84 | 240 |

Which statement is true?

Ⓐ Jones won the election with 65% of the total vote.

Ⓑ Smith won 75% of all votes from the South District.

Ⓒ Voters' choices are independent of the district in which they voted.

Ⓓ Support for Jones was proportionally higher in the South District than in the North District.

**Points Possible:** 1

**Content Cluster:** Understand independence and conditional probability, and use them to interpret data.

**Content Standard:** Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. *For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.★ (S.CP.4)*

**Depth of Knowledge:** Level 2
i. Retrieve information from a table, graph, or figure and use it to solve a problem requiring multiple steps
j. Translate between tables, graphs, words and symbolic notation

# Scoring Guidelines

Rationale for Option A: This is incorrect. The student may incorrectly think that if Jones wins the North District by 65% ($^{39}/_{60}$), then Jones wins the election. The student may only compare polling results for the two candidates in the North District.

Rationale for Option B: **Key** – The student correctly finds the percentage of votes for Smith, given they are from the South District by dividing the number of people who vote for Smith and are in the South District (135) by the total number of people in the South District (180) or $^{135}/_{180}$ = 75%.

Rationale for Option C: This is incorrect. The student may incorrectly think that if the percentages stating what percent of voters vote for each candidate in each district are different, then the voters' choices are independent of the district in which they live.

Rationale for Option D: This is incorrect. The student may incorrectly compare the number of votes in each district, 39 and 45, instead of comparing the ratios of the vote in each district, 65% ($^{39}/_{60}$), and 25% ($^{45}/_{80}$).

## Sample Response: 1 point

The table shows the voting results of a mayoral election in a small town between candidates Smith and Jones. The small town has two polling places, one in the North District and one in the South District.

| | Smith | Jones | Total |
|---|---|---|---|
| **North District** | 21 | 39 | 60 |
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Which statement is true?

Ⓐ Jones won the election with 65% of the total vote.

● Smith won 75% of all votes from the South District.

Ⓒ Voters' choices are independent of the district in which they voted.

Ⓓ Support for Jones was proportionally higher in the South District than in the North District.

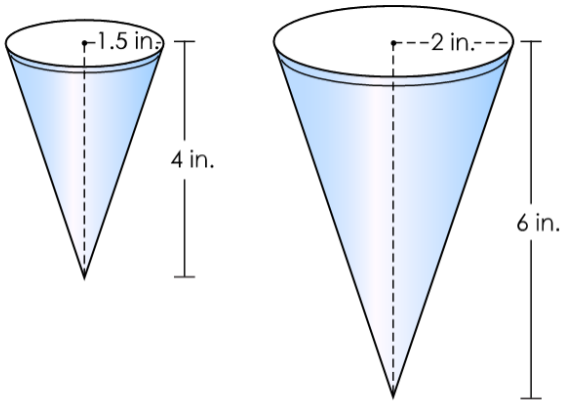
# Geometry
# Spring 2021 Item Release

# Question 4

# Question and Scoring Guidelines

# Question 4

A doctor's office is deciding between buying 2 different sized cups. The 2 cups are shown.




Before deciding which size cup to buy, they determine how much water each size cup can hold.

How much more water, in cubic inches, can the larger cup hold than the smaller cup? Round your answer to the nearest hundredth.

*cubic inches*

**Points Possible:** 1

**Content Cluster:** Explain volume formulas and use them to solve problems.

**Content Standard:** Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.★ *(G.GMD.3)*

**Depth of Knowledge:** Level 2
e. Compare and/or contrast figures or statements

# Scoring Guidelines

Exemplar Response

- 15.71 cubic inches

Other Correct Responses

- any value between 15.7 and 15.72 cubic inches, inclusive

For this item, a full-credit response includes:

- a correct value (1 point).

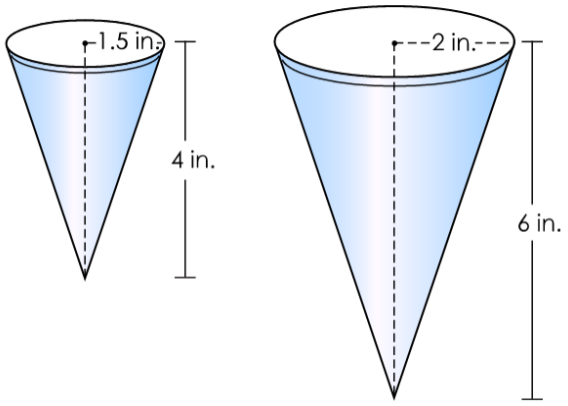
# Geometry
# Spring 2021 Item Release

# Question 4

# Sample Responses

## Sample Response: 1 point

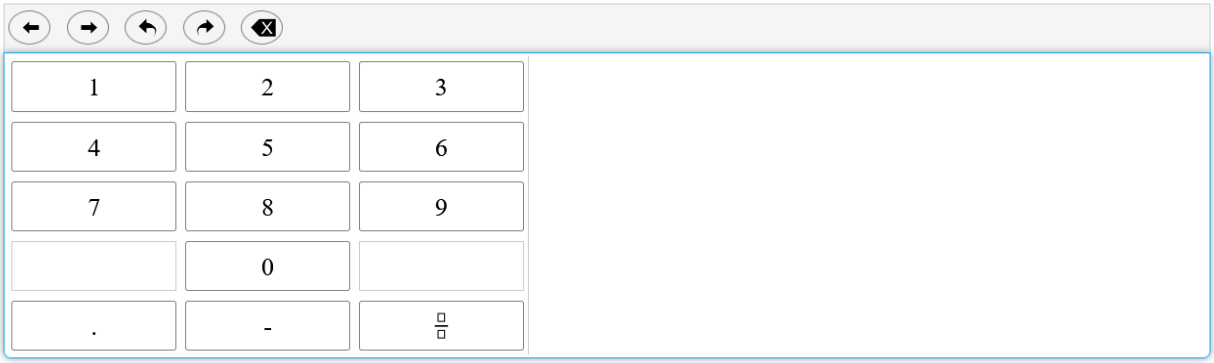
A doctor's office is deciding between buying 2 different sized cups. The 2 cups are shown.




Before deciding which size cup to buy, they determine how much water each size cup can hold.

How much more water, in cubic inches, can the larger cup hold than the smaller cup? Round your answer to the nearest hundredth.

15.7 *cubic inches*



**Notes on Scoring**

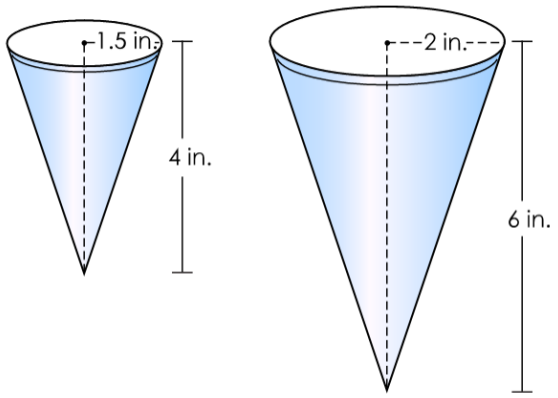
This response earns full credit (1 point) because it shows a correct difference in volumes of two cones, rounded to the nearest hundredth of a cubic inch.

To find how much more water the larger cup can hold than the smaller cup, the student may use the formula for the volume, $V = \frac{1}{3}\pi r^2 h$, where *r* is the radius of the base and *h* is the height of the cone, to find the difference between the volumes of the cones. The volume of the larger cone is $\frac{1}{3}\pi 2^2 \bullet 6 = 8\pi$ cubic inches , and the volume of the smaller cone is $\frac{1}{3}\pi 1.5^2 \bullet 4 = 3\pi$ cubic inches. The difference between the volumes is $8\pi - 3\pi = 5\pi$. When rounded to the nearest tenth, the answer is 15.7 cubic inches.

Answers between 15.7 and 15.72 are accepted to allow for minor differences in calculations of volumes.

## Sample Response: 1 point

A doctor's office is deciding between buying 2 different sized cups. The 2 cups are shown.




Before deciding which size cup to buy, they determine how much water each size cup can hold.

How much more water, in cubic inches, can the larger cup hold than the smaller cup? Round your answer to the nearest hundredth.

15.71 *cubic inches*

| | | |
|---|---|---|
| 1 | 2 | 3 |
| 4 | 5 | 6 |
| 7 | 8 | 9 |
| | 0 | |
| . | - | |

**Notes on Scoring**

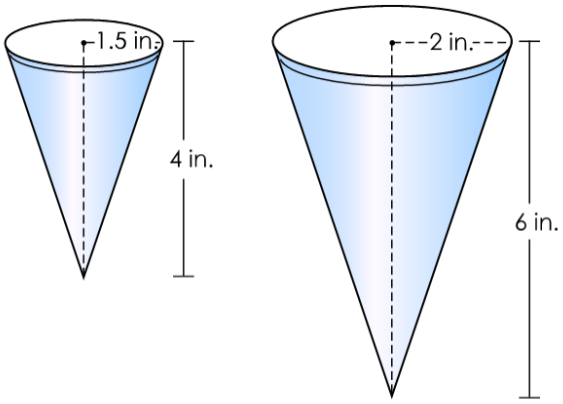
This response earns full credit (1 point) because it shows a correct difference in volumes of two cones, rounded to the nearest hundredth of a cubic inch.

To find how much more water the larger cup can hold than the smaller cup, the student may use the formula for volume, $V = \frac{1}{3}\pi r^2 h$, where *r* is the radius of the base and *h* is the height of the cone. The volume of the larger cone is $\frac{1}{3}\pi 2^2 \bullet 6 = 8\pi$ cubic inches, and the volume of the smaller cone is $\frac{1}{3}\pi 1.5^2 \bullet 4 = 3\pi$ cubic inches. The difference between the volumes is $8\pi - 3\pi = 5\pi$. When rounded to the nearest hundredths, the answer is 15.71 cubic inches.

Answers between 15.7 and 15.72 are accepted to allow for minor differences in calculations of volumes.

## Sample Response: 0 points

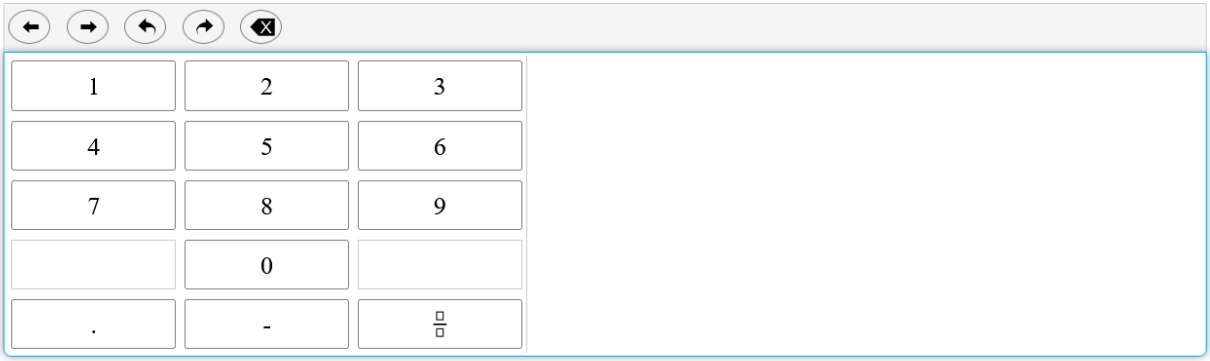
A doctor's office is deciding between buying 2 different sized cups. The 2 cups are shown.




Before deciding which size cup to buy, they determine how much water each size cup can hold.

How much more water, in cubic inches, can the larger cup hold than the smaller cup? Round your answer to the nearest hundredth.

46.83 *cubic inches*



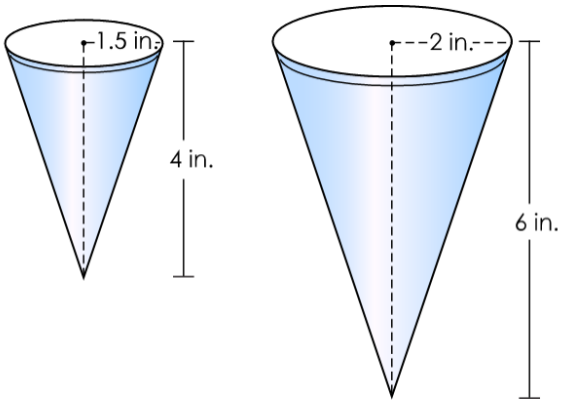
**Notes on Scoring**

This response earns no credit (0 points) because it shows an incorrect difference in volumes of two cones, rounded to the nearest hundredth of a cubic inch.

The student may realize that to find how much more water the larger cup can hold than the smaller cup, he or she needs to find the difference between the volumes of the cones. However, the student may confuse the formula for the cone, $V = \frac{1}{3}\pi r^2 h$, with the formula for the cylinder, $V = \pi r^2 h$, where *r* is the radius of the base, *h* is the height of the cone and $\pi \approx 3.14$. Then, the student may subtract the smaller volume from the larger volume to get 46.83 cubic inches.

## Sample Response: 0 points

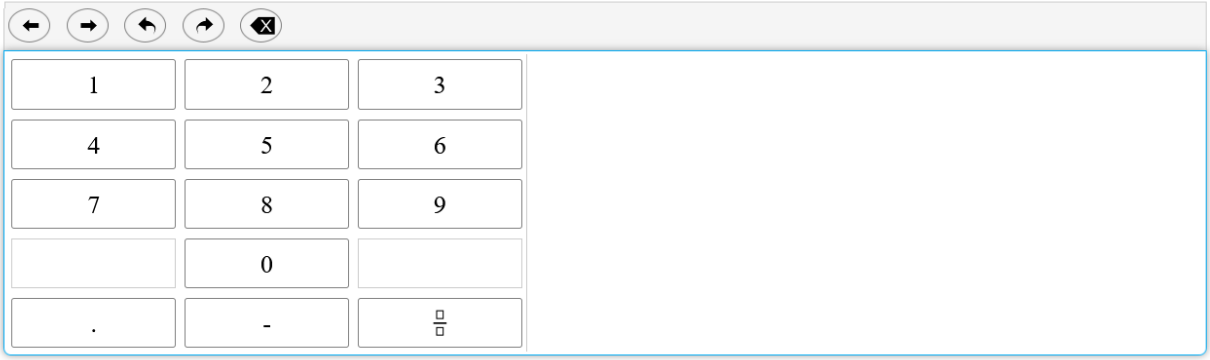
A doctor's office is deciding between buying 2 different sized cups. The 2 cups are shown.




Before deciding which size cup to buy, they determine how much water each size cup can hold.

How much more water, in cubic inches, can the larger cup hold than the smaller cup? Round your answer to the nearest hundredth.

6.28 *cubic inches*



**Notes on Scoring**

This response earns no credit (0 points) because it shows an incorrect difference in volumes of two cones, rounded to the nearest hundredth of a cubic inch.

The student may realize that to find how much more water the larger cup can hold than the smaller cup, he or she needs to find the difference between the volumes of the cones. However, the student may confuse the formula for the cone, $V = \frac{1}{3}\pi r^2 h$, with the incorrect formula, $V = \frac{1}{3}\pi r h$, where *r* is the radius of the base, *h* is the height of the cone and $\pi \approx 3.14$. Then, the student may subtract the smaller volume from the larger volume to get 6.28 cubic inches.

# Geometry
# Spring 2021 Item Release

# Question 5

# Question and Scoring Guidelines

# Question 5

This item has **two** parts.

Regis leans a 10-foot ladder against a wall. The base of the ladder makes a 65° angle with the ground.

**Part A.** What is the distance, in feet, from the base of the ladder to the base of the wall? Round to the nearest tenth of a foot.

*feet*

| | | |
|---|---|---|
| 1 | 2 | 3 |
| 4 | 5 | 6 |
| 7 | 8 | 9 |
| | 0 | |
| . | - | ▫/▫ |

**Part B.** Regis needs to move the ladder so that it reaches a window 9.6 feet above the ground.

How many feet **closer** to the building does he need to move the base of the ladder?

*feet*

| | | |
|---|---|---|
| 1 | 2 | 3 |
| 4 | 5 | 6 |
| 7 | 8 | 9 |
| | 0 | |
| . | - | ▫/▫ |

**Points Possible:** 2

**Content Cluster:** Define trigonometric ratios, and solve problems involving right triangles.

**Content Standard:** Solve problems involving right triangles.
*a.* Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems if one of the two acute angles and a side length are given.★ *(G.SRT.8)*

**Depth of Knowledge:** Level 3
e. Use concepts to solve non-routine problems
f. Perform a procedure with multiple steps and multiple decision points
m. Translate between a problem situation and symbolic notation that is not a direct translation

# Scoring Guidelines

For full credit (2 points), the student's response satisfies both the bullets below.

- The student enters a value between 4.2 and 4.23 feet, providing evidence of the ability to use trigonometric ratios to solve problems.
- The student enters 1.4 or a positive value that is 2.8 less than Part A, providing evidence of the ability to use the Pythagorean Theorem to solve problems.

Exemplar Response

- **Part A**
  4.2 feet, or any value between 4.2 and 4.23, inclusive

  **Part B**
  1.4 feet, or any value 2.8 less than Part A given Part B is greater than 0, or any value between 1.4 and 1.43, inclusive

For partial credit (1 point), the student's response satisfies one of the bullets.

- The student enters a value between 4.2 and 4.23 feet, providing evidence of the ability to use trigonometric ratios to solve problems.
- The student enters 1.4 or a positive value that is 2.8 less than Part A, providing evidence of the ability to use the Pythagorean Theorem to solve problems.

Exemplar Response

- **Part A**
  4.2 feet

  **Part B**
  2.8 feet

# Geometry
# Spring 2021 Item Release

# Question 5

# Sample Responses

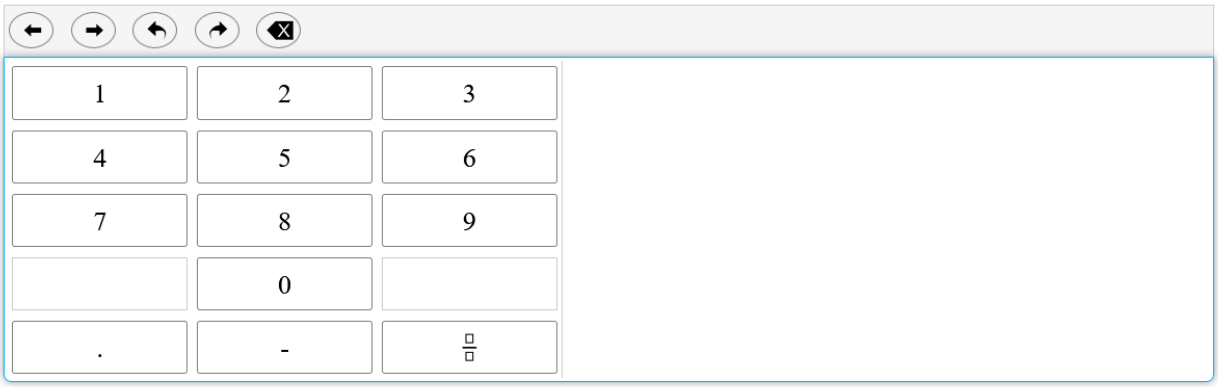
## Sample Response: 2 points

This item has **two** parts.

Regis leans a 10-foot ladder against a wall. The base of the ladder makes a 65° angle with the ground.

**Part A.** What is the distance, in feet, from the base of the ladder to the base of the wall? Round to the nearest tenth of a foot.

4.2 *feet*



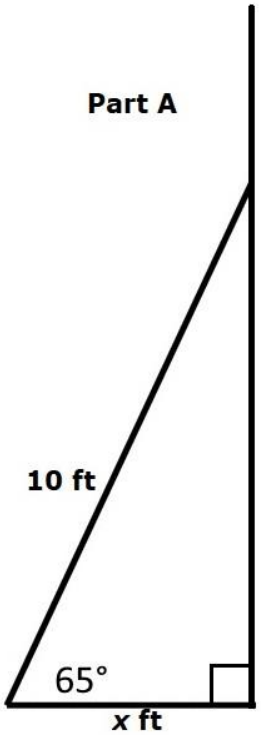

**Part B.** Regis needs to move the ladder so that it reaches a window 9.6 feet above the ground.

How many feet **closer** to the building does he need to move the base of the ladder?

1.4 *feet*

**Notes on Scoring**

This response earns full credit (2 points) because it shows the correct distance in Part A and the correct number of feet in Part B.

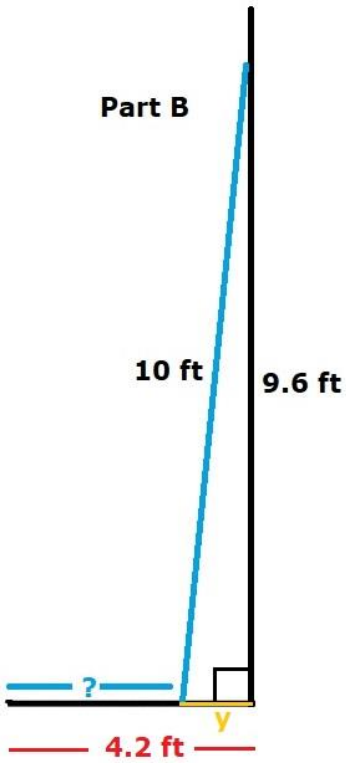



A leaning ladder, a vertical wall and the ground make a right triangle, where a hypotenuse is the ladder and $65°$ angle is between the hypotenuse and the adjacent horizontal side.

Since the length of the hypotenuse is 10 feet and the unknown side adjacent to the angle of $65°$ is x, the student understands the right triangle definition of cosine can be used to find that unknown side. In right triangles, the cosine of an acute angle is the ratio of the length of the adjacent side to the length of hypotenuse. Based on this definition, $\cos 65° = \frac{x}{10}$. After multiplying both sides of this equation by 10, $x = 10 \bullet \cos 65°$, or $x = 4.2$, when rounded to the nearest tenth of a foot.

In Part A, values between 4.2 and 4.23 are acceptable to allow for minor differences in calculations when using trigonometric ratios.

**Notes on Scoring (continued)**




When Regis moves the ladder so that it can reach the height of 9.6 feet along the vertical wall, the ladder, the wall and the ground form a different right triangle. The length of the hypotenuse remains 10 feet, because the length of the ladder does not change, but the distance between the base of the ladder and the wall changes. This unknown side length, $y$, of the new right triangle can be found using Pythagorean Theorem as:

$10^2 = 9.6^2 + y^2$

$y^2 = 10^2 - 9.62^2$

$y^2 = 7.84$

$y = 2.8$

To calculate how many feet closer to the building Regis needs to move the base of the ladder, find the difference between the answer in Part A and 2.8, or $4.2 - 2.8 = 1.4$ feet.

In Part B, any positive value that is 2.8 less than the answer in Part A is acceptable.

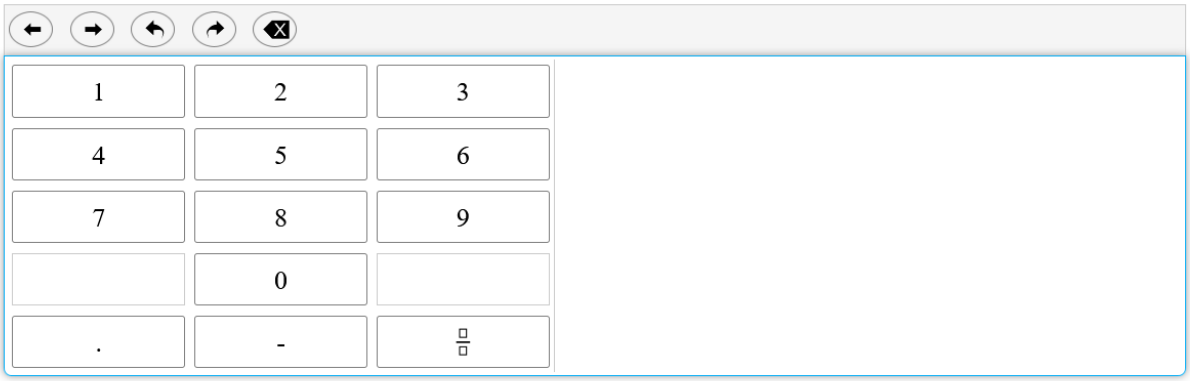
## Sample Response: 2 points

This item has **two** parts.

Regis leans a 10-foot ladder against a wall. The base of the ladder makes a 65° angle with the ground.

**Part A.** What is the distance, in feet, from the base of the ladder to the base of the wall? Round to the nearest tenth of a foot.

4.23 *feet*




**Part B.** Regis needs to move the ladder so that it reaches a window 9.6 feet above the ground.

How many feet **closer** to the building does he need to move the base of the ladder?

1.43 *feet*

**Notes on Scoring**

This response earns full credit (2 points) because it shows the correct distance in Part A and the correct number of feet in Part B.

A leaning ladder, a vertical wall and the ground make a right triangle, where a hypotenuse is the ladder and $65°$ angle is between the hypotenuse and the adjacent horizontal side.

Since the length of hypotenuse is 10 feet and the unknown side adjacent to the angle of $65°$ is x, the student understands the right triangle definition of cosine can be used to find that unknown side. In right triangles, the cosine of an acute angle is the ratio of the length of the adjacent side to the length of the hypotenuse. Based on this definition, $\cos 65° = \frac{x}{10}$. After multiplying both sides of this equation by 10, $x = 10 \bullet \cos 65°$, or $x = 4.2$, when rounded to the nearest tenth of a foot.

In Part A, values between 4.2 and 4.23 are acceptable to allow for minor differences in calculations when using trigonometric ratios.

When Regis moves the ladder so that it can reach the height of 9.6 feet along the vertical wall, the ladder, the wall and the ground form a different right triangle. The length of the hypotenuse remains 10 feet, because the length of the ladder does not change, but the distance between the base of the ladder and the wall changes. This unknown side length, $y$, of the new right triangle can be found using Pythagorean Theorem as:

$$10^2 = 9.6^2 + y^2$$
$$y^2 = 10^2 - 9.6^2$$
$$y^2 = 7.84$$
$$y = 2.8$$

To calculate how many feet closer to the building Regis needs to move the base of the ladder, find the difference between the answer in Part A and 2.8, or $4.23 - 2.8 = 1.43$ feet.

In Part B, any positive value that is 2.8 less than the answer in Part A is acceptable.

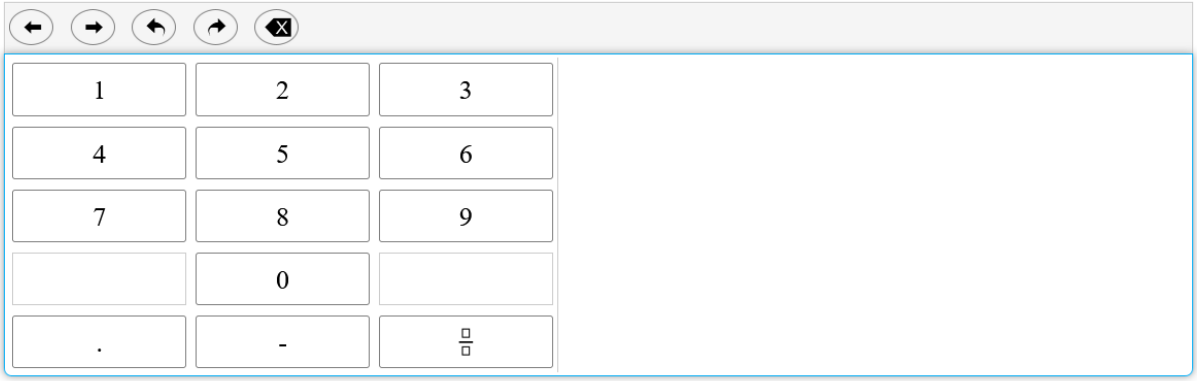
## Sample Response: 1 point

This item has **two** parts.

Regis leans a 10-foot ladder against a wall. The base of the ladder makes a 65° angle with the ground.

**Part A.** What is the distance, in feet, from the base of the ladder to the base of the wall? Round to the nearest tenth of a foot.

4.2 *feet*




**Part B.** Regis needs to move the ladder so that it reaches a window 9.6 feet above the ground.

How many feet **closer** to the building does he need to move the base of the ladder?

2 *feet*

**Notes on Scoring**

This response earns partial credit (1 point) because it shows the correct distance in Part A and the incorrect number of feet in Part B. A leaning ladder, a vertical wall and the ground make a right triangle, where a hypotenuse is the ladder and $65°$ angle is between the hypotenuse and the adjacent side.

Since the length of hypotenuse is 10 feet and the unknown side adjacent to the angle of $65°$ is x, the student understands the right triangle definition of cosine can be used to find that unknown side. In right triangles, the cosine of an acute angle is the ratio of the length of the adjacent side to the length of the hypotenuse. Based on this definition, $\cos 65° = \frac{x}{10}$. After multiplying both sides of this equation by 10, $x = 10 \bullet \cos 65°$, or $x = 4.2$, when rounded to the nearest tenth of a foot.

When Regis moves the ladder so that it can reach the height of 9.6 feet along the vertical wall, the ladder, the wall and the ground still form a right triangle. The length of the hypotenuse remains 10 feet, because the length of the ladder does not change, but the distance between the base of the ladder and the wall changes. This unknown side length, $y$, of the new right triangle can be found using Pythagorean Theorem as:

$$10^2 = 9.6^2 + y^2$$
$$y^2 = 10^2 - 9.6^2$$
$$y^2 = 7.84$$
$$y = 2.8$$

To calculate how many feet closer to the building Regis needs to move the base of the ladder, the student may subtract 2.8 from 4.2 to find 1.4 feet and then incorrectly rounds that value to the next nearest foot.

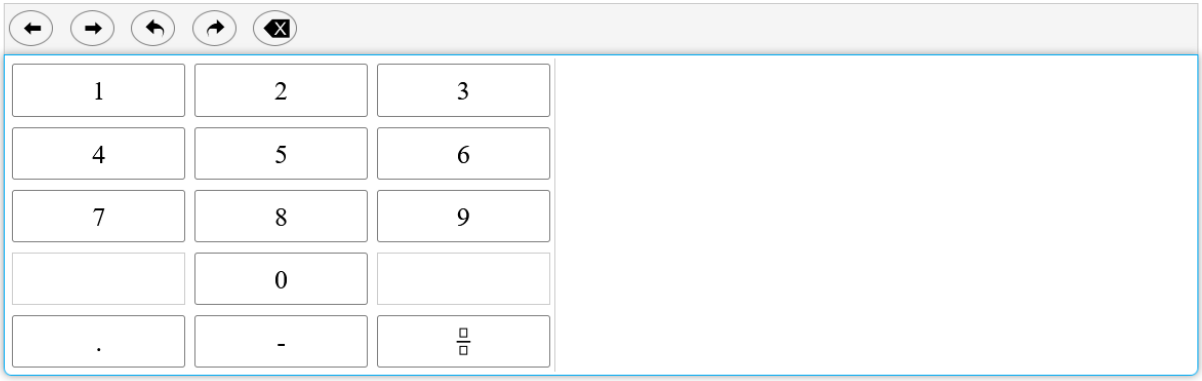
## Sample Response: 1 point

This item has **two** parts.

Regis leans a 10-foot ladder against a wall. The base of the ladder makes a 65° angle with the ground.

**Part A.** What is the distance, in feet, from the base of the ladder to the base of the wall? Round to the nearest tenth of a foot.

4.2 *feet*




**Part B.** Regis needs to move the ladder so that it reaches a window 9.6 feet above the ground.

How many feet **closer** to the building does he need to move the base of the ladder?

2.8 *feet*

**Notes on Scoring**

This response earns partial credit (1 point) because it shows the correct distance in Part A and incorrect number of feet in Part B.

A leaning ladder, a vertical wall and the ground make a right triangle, where a hypotenuse is the ladder and $65°$ angle is between the hypotenuse and the adjacent side.

Since the length of the hypotenuse is 10 feet and the unknown side adjacent to the angle of $65°$ is x, the student understands the right triangle definition of cosine can be used to find that unknown side. In right triangles, the cosine of an acute angle is the ratio of the length of the adjacent side to the length of the hypotenuse. Based on this definition, $\cos 65° = \frac{x}{10}$. After multiplying both sides of this equation by 10, $x = 10 \bullet \cos 65°$, or $x = 4.2$, when rounded to the nearest tenth of a foot.

When Regis moves the ladder so that it can reach the height of 9.6 feet along the vertical wall, the ladder, the wall and the ground still form a right triangle. The length of the hypotenuse remains as 10 feet, because the length of the ladder does not change, but the distance between the base of the ladder and the wall changes. This unknown side length, $y$, of the new right triangle can be found using Pythagorean Theorem as:

$$10^2 = 9.6^2 + y^2$$
$$y^2 = 10^2 - 9.62^2$$
$$y^2 = 7.84$$
$$y = 2.8$$

The student may forget to find the difference between 4.2 and 2.8 and instead uses 2.8 as the answer for Part B.

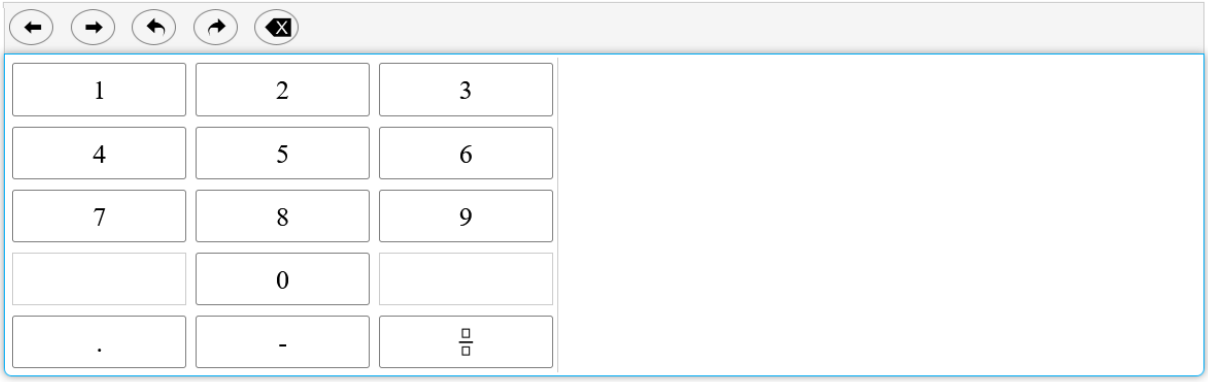
## Sample Response: 0 points

This item has **two** parts.

Regis leans a 10-foot ladder against a wall. The base of the ladder makes a 65° angle with the ground.

**Part A.** What is the distance, in feet, from the base of the ladder to the base of the wall? Round to the nearest tenth of a foot.

6.5 *feet*




**Part B.** Regis needs to move the ladder so that it reaches a window 9.6 feet above the ground.

How many feet **closer** to the building does he need to move the base of the ladder?

3.1 *feet*

**Notes on Scoring**

This response earns no credit (0 points) because it shows an incorrect distance in Part A and the incorrect number of feet in Part B.
The student may incorrectly divide 65 by 10 to get 6.5 for the answer for Part A and then subtracts this result from the height of the window, or 9.6 – 6.5 = 3.1, to determine the answer in Part B.

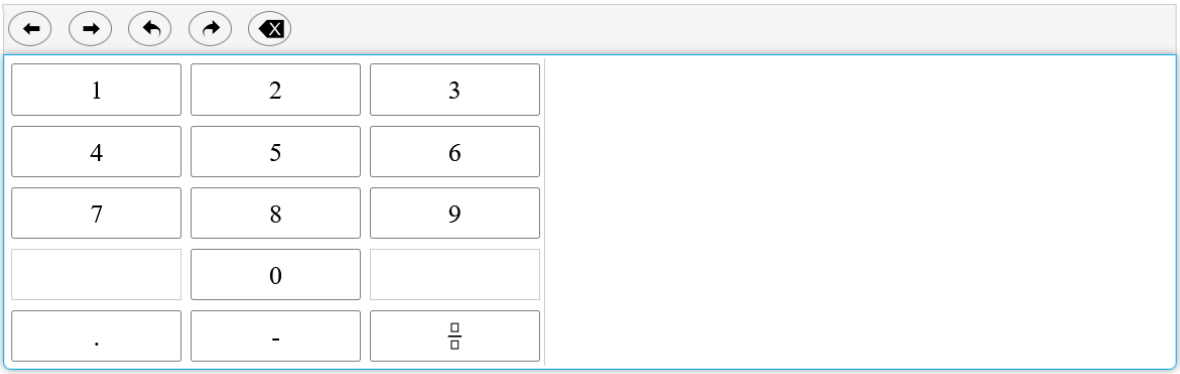
## Sample Response: 0 points

This item has **two** parts.

Regis leans a 10-foot ladder against a wall. The base of the ladder makes a 65° angle with the ground.

**Part A.** What is the distance, in feet, from the base of the ladder to the base of the wall? Round to the nearest tenth of a foot.

5 *feet*




**Part B.** Regis needs to move the ladder so that it reaches a window 9.6 feet above the ground.

How many feet **closer** to the building does he need to move the base of the ladder?

2 *feet*

**Notes on Scoring**

This response earns no credit (0 points) because it shows an incorrect distance in Part A and an incorrect number of feet in Part B.

The student may incorrectly use the angle of $60°$ instead of $65°$ when calculating the side adjacent to the given angle as $\cos 60° = \frac{x}{10}$. Next the student may multiply both sides of this equation by 10 to get $x = 10 \bullet \cos 60°$, or $x = 5$ feet.

When Regis moves the ladder so that it can reach the height of 9.6 feet along the vertical wall, the ladder, the wall and the ground still form a right triangle. The length of the hypotenuse remains 10 feet, because the length of the ladder does not change, but the distance between the base of the ladder and the wall changes. This unknown side length, $y$, of the new right triangle can be found using Pythagorean Theorem as

$$10^2 = 9.6^2 + y^2$$
$$y^2 = 10^2 - 9.6^2$$
$$y^2 = 7.84$$
$$y = 2.8$$

To calculate the answer for Part B, or how many feet closer to the building Regis needs to move the base of the ladder, the student may subtract 2.8 from 5 to find 2.2 feet and then incorrectly rounds that value to the next nearest whole foot, or 2.

# Geometry
# Spring 2021 Item Release

# Question 11

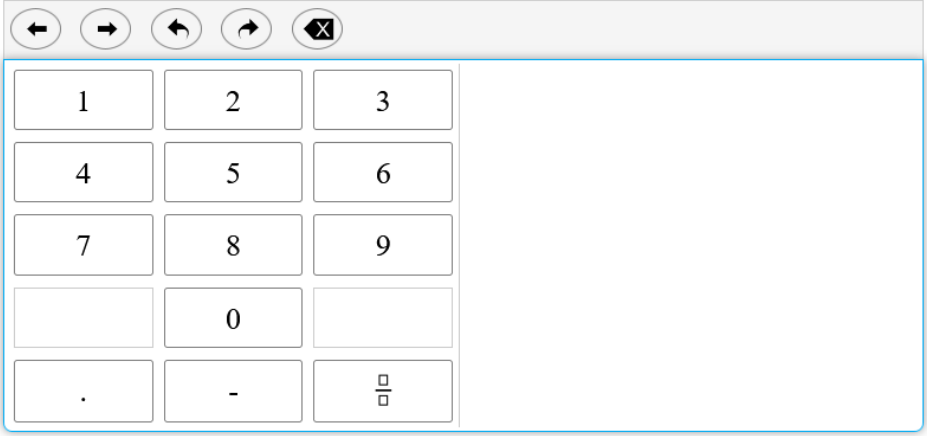
# Question and Scoring Guidelines

# Question 11

Probabilities for two events, event $A$ and event $B$, are given.

$P(A \text{ and } B) = 0.14$
$P(B) = 0.4$

What is the probability of $A$ given $B$?



**Points Possible:** 1

**Content Cluster:** Understand independence and conditional probability, and use them to interpret data.

**Content Standard:** Understand the conditional probability of A given B as $P(A \text{ and } B)/P(B)$, and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B.★ *(S.CP.3)*

**Depth of Knowledge:** Level 2
k. Make direct translations between problem situations and symbolic notation
l. Select a procedure according to criteria and perform it

# Scoring Guidelines

Exemplar Response

- 0.35

Other Correct Responses

- any equivalent value

For this item, a full-credit response includes:

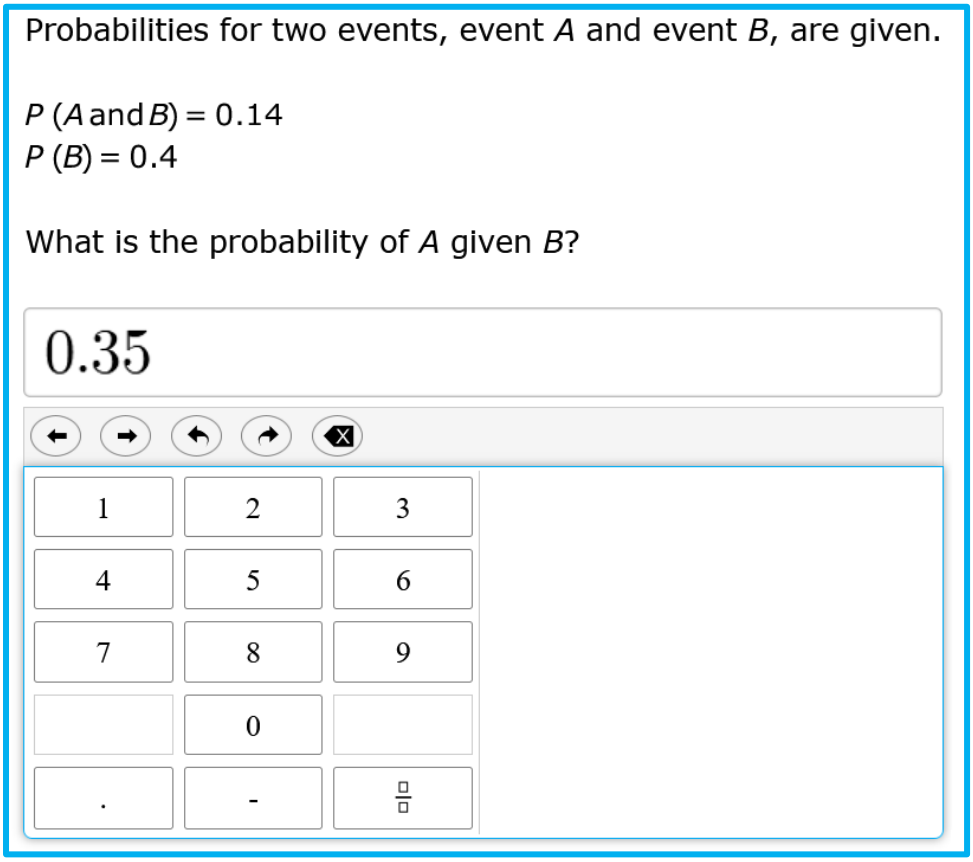
- the correct value (1 point).

# Geometry
# Spring 2021 Item Release

# Question 11

# Sample Responses

## Sample Response: 1 point

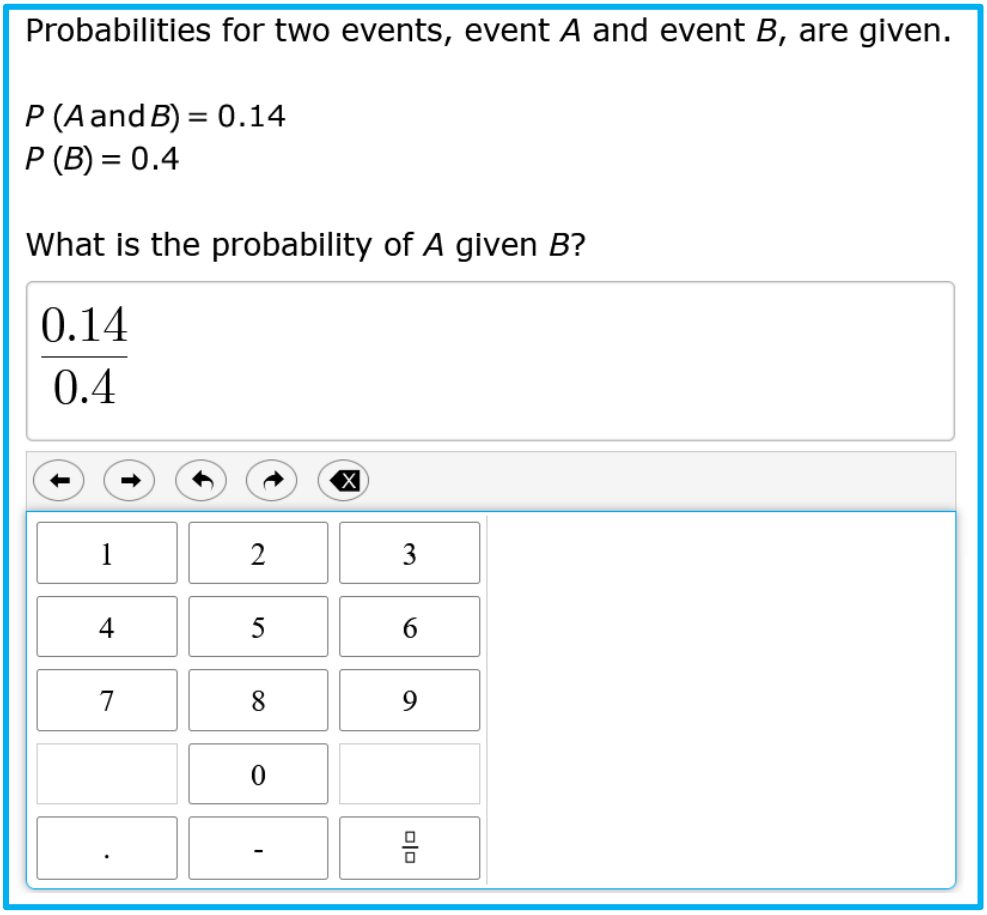



**Notes on Scoring**

This response earns full credit (1 point) because it shows the correct conditional probability of event *A* given event *B*.

The student understands that the probability of *A* given *B* is equal to *P*(*A* and *B*) divided by *P*(*B*). In this situation, since *P*(*A* and *B*) = 0.14 and *P*(*B*) = 0.4, then $P(A \mid B) = \frac{0.14}{0.4} = 0.35$.

## Sample Response: 1 point

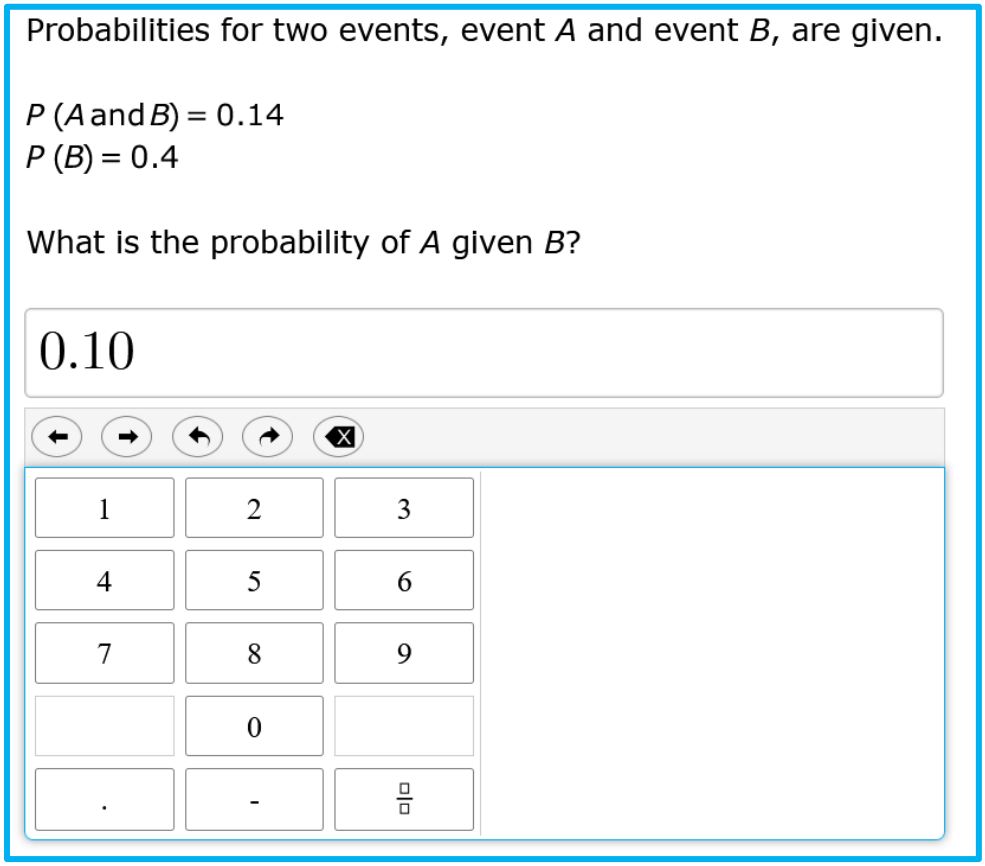



**Notes on Scoring**

This response earns full credit (1 point) because it shows the correct conditional probability of event *A* given event *B*.

The student understands that the probability of *A* given *B* is equal to *P*(*A* and *B*) divided by *P*(*B*). In this situation, since *P*(*A* and *B*) = 0.14 and *P*(*B*) = 0.4, then $P(A \mid B) = \frac{0.14}{0.4}$.

## Sample Response: 0 points

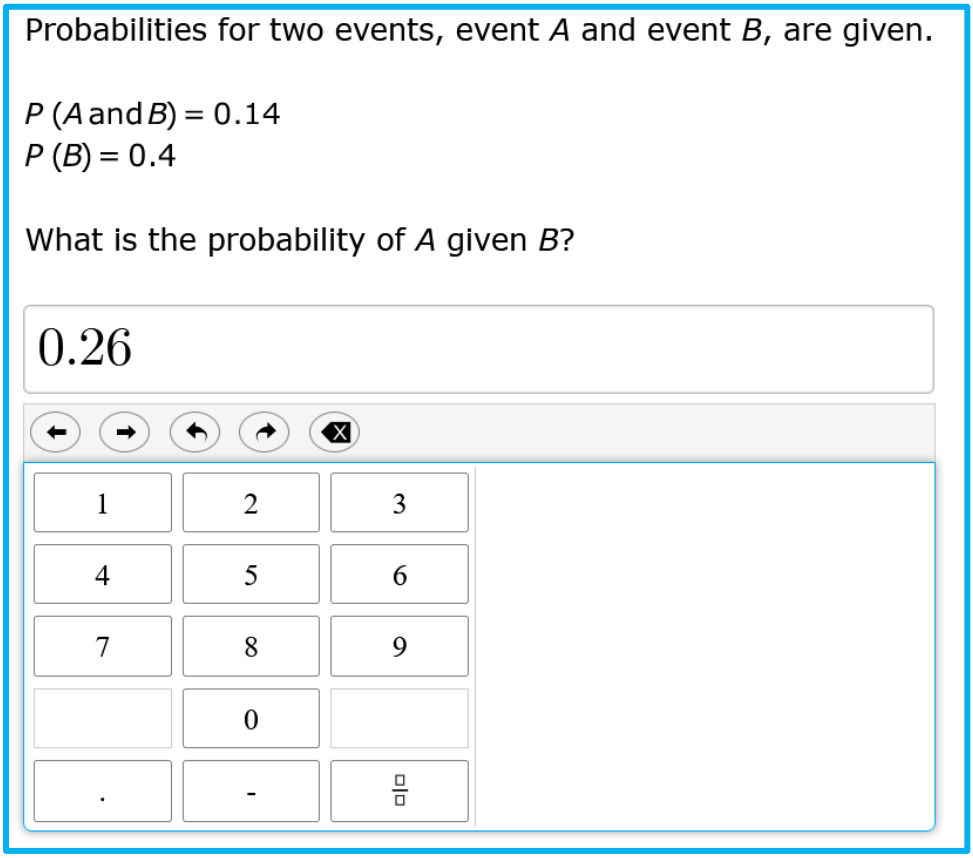



**Notes on Scoring**

This response earns no credit (0 points) because it shows an incorrect conditional probability of event *A* given event *B*.

The student may misinterpret 0.4 as 0.04 and subtracts *P*(*B*) from *P*(*A* and *B*) to get 0.14 – 0.04 = 0.10.

## Sample Response: 0 points




**Notes on Scoring**

This response earns no credit (0 points) because it shows an incorrect conditional probability of event *A* given event *B*.

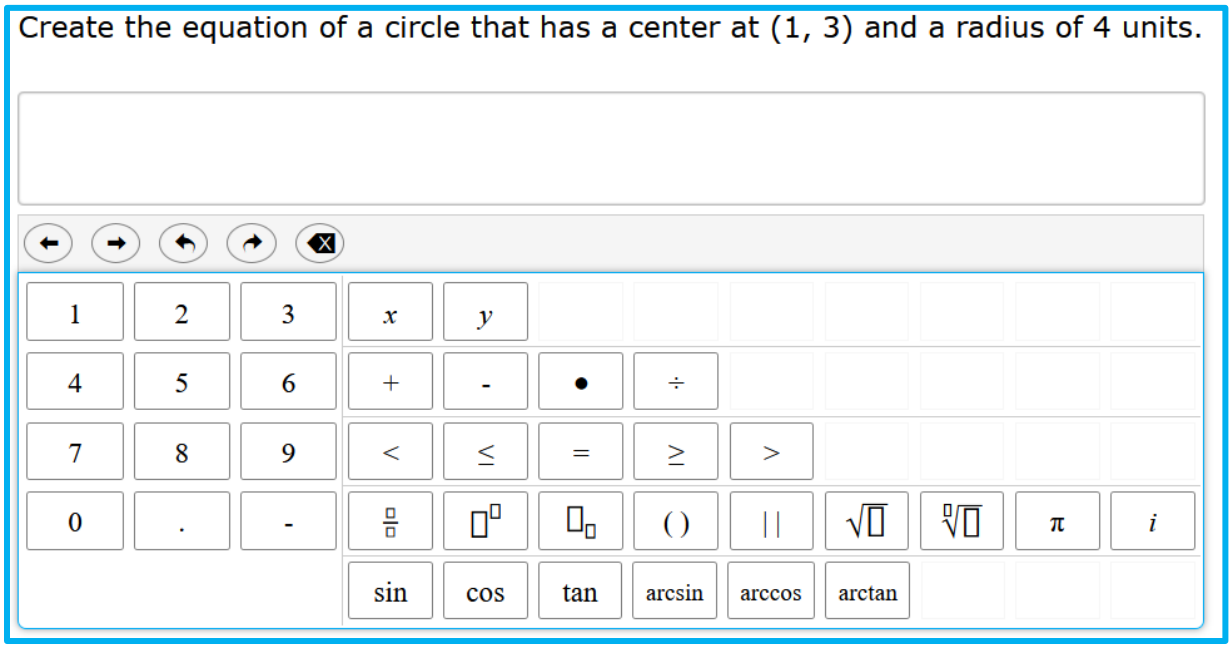
The student may subtract *P*(*A* and *B*) from the probability *P*(*B*) to get 0.4 – 0.14 = 0.26.

# Geometry
# Spring 2021 Item Release

# Question 17

# Question and Scoring Guidelines

# Question 17




**Points Possible:** 1

**Content Cluster:** Translate between the geometric description and the equation for a conic section.

**Content Standard:** Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation. *(G.GPE.1)*

**Depth of Knowledge:** Level 1
a. Recall, observe, or recognize a fact, definition, term, or property
b. Apply/compute a well-known algorithm (e.g., sum, quotient)

# Scoring Guidelines

Exemplar Response

- $(x - 1)^2 + (y - 3)^2 = 4^2$

Other Correct Responses

- any equivalent equation

For this item, a full-credit response includes:

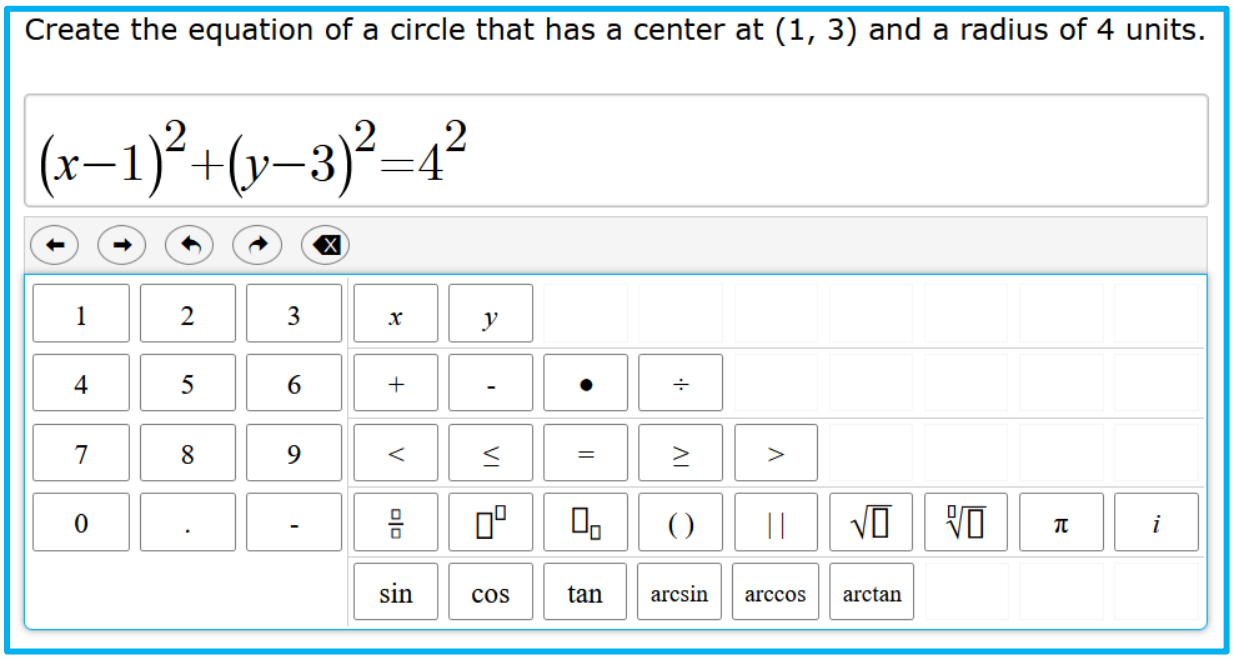
- a correct equation (1 point).

# Geometry
# Spring 2021 Item Release

# Question 17

# Sample Responses

## Sample Response: 1 point

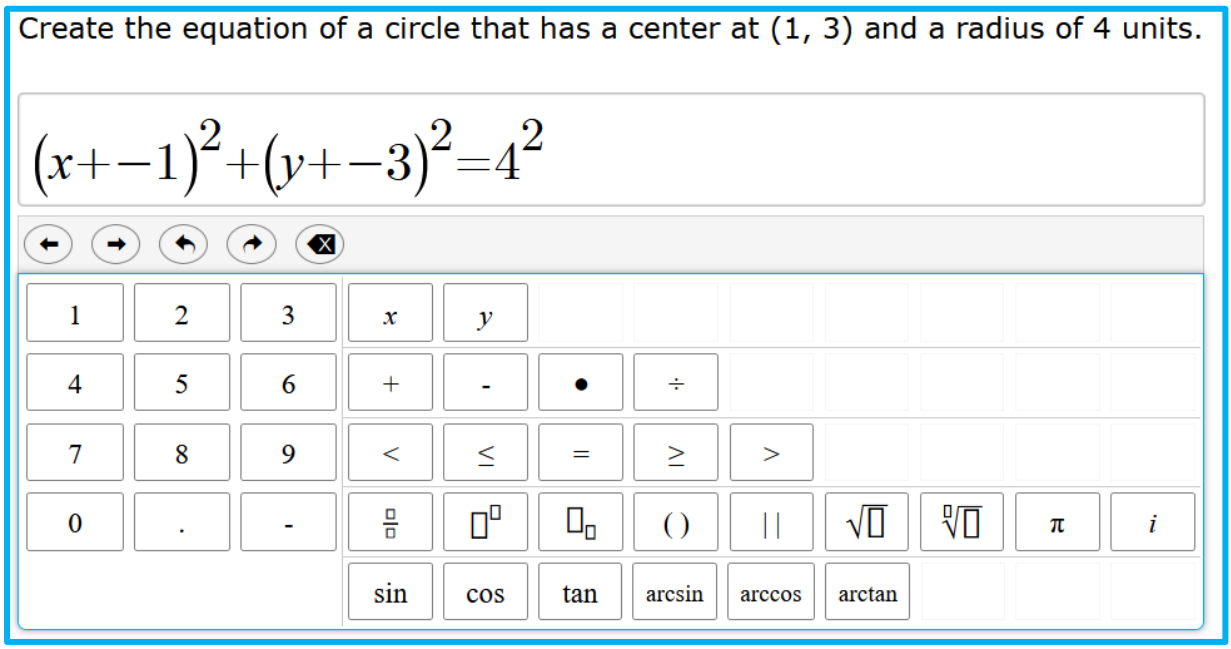



**Notes on Scoring**

This response earns full credit (1 point) because it shows the correct equation of the circle in center-radius form.

The student understands that, on the coordinate plane, the center-radius form for the equation of a circle with center (*h*, *k*) and radius *r* is $(x - h)^2 + (y - k)^2 = r^2$. The given circle has a center at (1, 3) and a radius of 4 units. By substituting $h = 1$, $k = 3$ and $r = 4$ in the center-radius form for *h*, *k* and *r*, respectively, the equation of the circle is $(x - 1)^2 + (y - 3)^2 = 4^2$.

## Sample Response: 1 point




**Notes on Scoring**

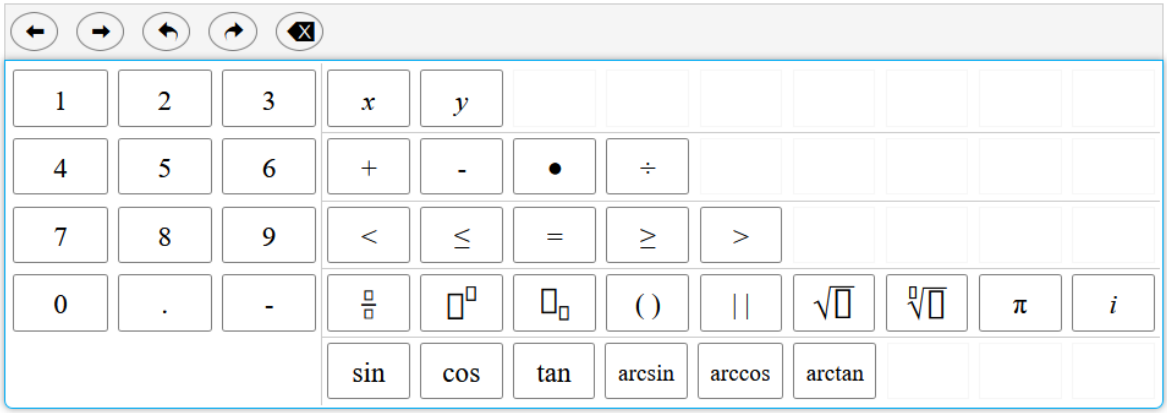
This response earns full credit (1 point) because it shows the correct equation of the circle in center-radius form.

On the coordinate plane, the center-radius form for the equation of a circle with center (*h*, *k*) and radius *r* is $(x - h)^2 + (y - k)^2 = r^2$. The given circle has a center at (1, 3) and a radius of 4 units. The student substitutes $h = 1$, $k = 3$ and $r = 4$ in the center-radius form for *h*, *k* and *r*, respectively, the equation of the circle is $(x - 1)^2 + (y - 3)^2 = 4^2$, which is equivalent to $(x + -1)^2 + (y + -3)^2 = 4^2$.

## Sample Response: 0 points

Create the equation of a circle that has a center at (1, 3) and a radius of 4 units.

$(x+1)^2+(y+3)^2=4^2$




**Notes on Scoring**

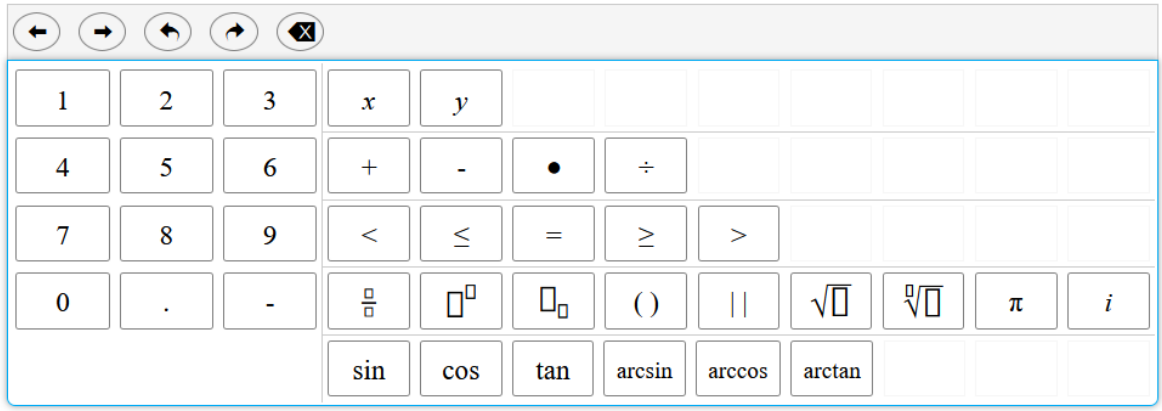
This response earns no credit (0 points) because it shows an incorrect equation of the circle in center-radius form.

The student may confuse the center-radius form for the equation of a circle $(x - h)^2 + (y - k)^2 = r^2$ with $(x + h)^2 + (y + k)^2 = r^2$ to get an incorrect equation $(x + 1)^2 + (y + 3)^2 = 4^2$.

## Sample Response: 0 points

Create the equation of a circle that has a center at (1, 3) and a radius of 4 units.

$(x-1)^2+(y-3)^2=4$




**Notes on Scoring**

This response earns no credit (0 points) because it shows an incorrect equation of the circle in center-radius form.

On the coordinate plane, the center-radius form for the equation of a circle with center (*h*, *k*) and radius *r* is $(x - h)^2 + (y - k)^2 = r^2$. The given circle has a center at (1, 3) and a radius of 4 units. The student may correctly substitute $h = 1$, $k = 3$ and $r = 4$ in the center-radius form for *h*, *k* and *r*, respectively but forget to square 4 as $(x - 1)^2 + (y - 3)^2 = 4$.

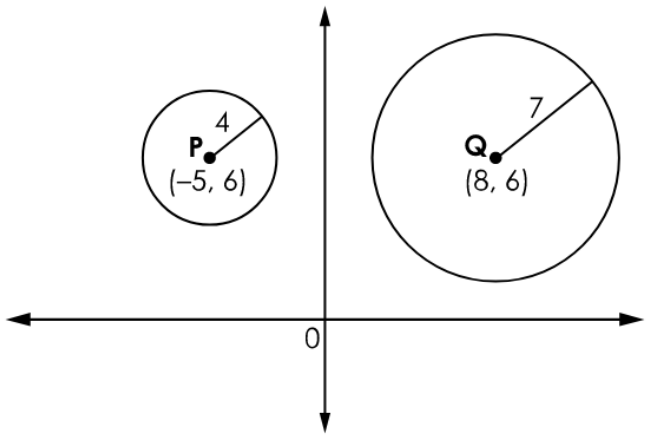
# Geometry
# Spring 2021 Item Release

# Question 20

# Question and Scoring Guidelines

# Question 20

Consider the two circles shown.




To show that circle P is similar to circle Q, circle P is translated $t$ units to the right. The image is then dilated about its center by a scale factor of $s$.

What are the values of $t$ and $s$?

$t =$

$s =$

**Points Possible:** 1

**Content Cluster:** Understand and apply theorems about circles.

**Content Standard:** Prove that all circles are similar using transformational arguments. *(G.C.1)*

**Depth of Knowledge:** Level 2
i. Retrieve information from a table, graph, or figure and use it to solve a problem requiring multiple steps
k. Make direct translations between problem situations and symbolic notation

# Scoring Guidelines

Exemplar Response

- $t = 13$
  $s = 1.75$

Other Correct Responses

- any equivalent value

For this item, a full-credit response includes:

- the correct values (1 point).

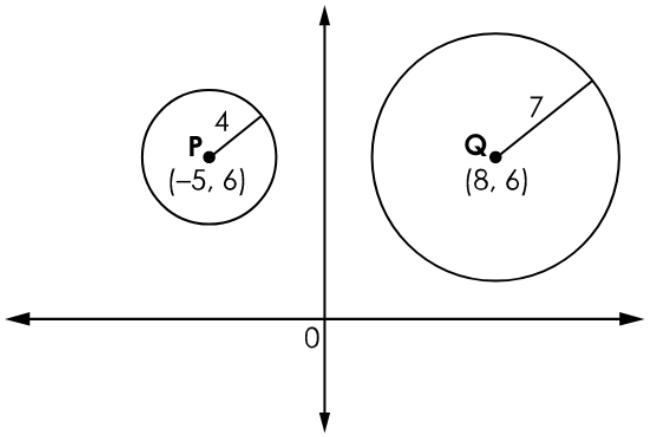
# Geometry
# Spring 2021 Item Release

# Question 20

# Sample Responses

## Sample Response: 1 point

Consider the two circles shown.




To show that circle P is similar to circle Q, circle P is translated $t$ units to the right. The image is then dilated about its center by a scale factor of $s$.

What are the values of $t$ and $s$?

$t =$ 13

$s =$ 1.75

| | | |
|---|---|---|
| 1 | 2 | 3 |
| 4 | 5 | 6 |
| 7 | 8 | 9 |
| | 0 | |
| . | - | $\frac{\square}{\square}$ |

**Notes on Scoring**

This response earns full credit (1 point) because it shows the correct values for units of translation and for the scale factor to prove a similarity of two circles.

To show that circle P and circle Q are similar, show that circle P can overlay circle Q by applying a sequence of transformations—a translation *t* units and a dilation by the scale factor *s* about the center of the translated image of circle P.

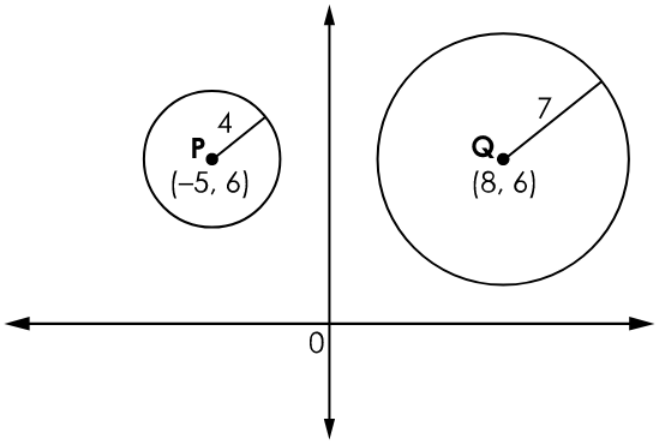
To get the centers of the circles to coincide, the student translates circle P to the right 13 units. This value can be calculated by subtracting the x-coordinates of two centers as, $t = 8 - (-5) = 13$. So, $t = 13$ units.

To get the circles to completely overlay, the student dilates the image of circle P by a scale factor $s = \frac{radius\ of\ circle\ Q}{radius\ of\ circle\ P}$.

The radius of circle P is 4 and the radius of circle Q is 7. The two circles entirely coincide when the scale factor $s = \frac{7}{4}$ or $s = 1.75$.

**Sample Response: 1 point**

Consider the two circles shown.




To show that circle P is similar to circle Q, circle P is translated $t$ units to the right. The image is then dilated about its center by a scale factor of $s$.

What are the values of $t$ and $s$?

$t =$ 13

$s =$ $\frac{7}{4}$

| | | |
|---|---|---|
| 1 | 2 | 3 |
| 4 | 5 | 6 |
| 7 | 8 | 9 |
| | 0 | |
| . | - | $\frac{\square}{\square}$ |

**Notes on Scoring**

This response earns full credit (1 point) because it shows the correct values for units of translation and for the scale factor to prove a similarity of two circles.

To show that circle P and circle Q are similar, show that circle P can overlay circle Q by applying a sequence of transformations—a translation by *t* units and a dilation by the scale factor *s* about the center of the translated image of circle P.

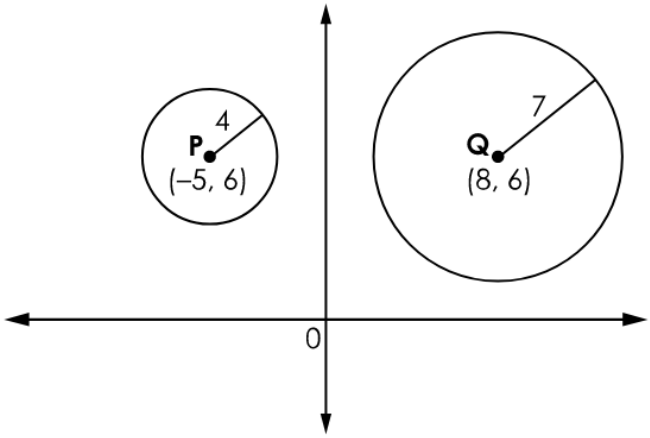
To get the centers of the circle to coincide, the student translates circle P to the right 13 units. This value can be calculated by subtracting the x-coordinates of two centers as, $t = 8 - (-5) = 13$. So, $t = 13$ units.

To get the circles to completely overlay, the student dilates circle P by a scale factor $s = \frac{radius\ of\ circle\ Q}{radius\ of\ circle\ P}$.

The radius of circle P is 4 and the radius of circle Q is 7. The two circles entirely coincide when circle P is enlarged by the scale factor $s = \frac{7}{4}$.

**Sample Response: 0 points**

Consider the two circles shown.




To show that circle P is similar to circle Q, circle P is translated $t$ units to the right. The image is then dilated about its center by a scale factor of $s$.

What are the values of $t$ and $s$?

$t =$ 13

$s =$ 3

| | | |
|---|---|---|
| 1 | 2 | 3 |
| 4 | 5 | 6 |
| 7 | 8 | 9 |
| | 0 | |
| . | - | $\frac{\square}{\square}$ |

**Notes on Scoring**

This response earns no credit (0 points) because it shows the correct values for units of translation and an incorrect scale factor to prove a similarity of two circles.

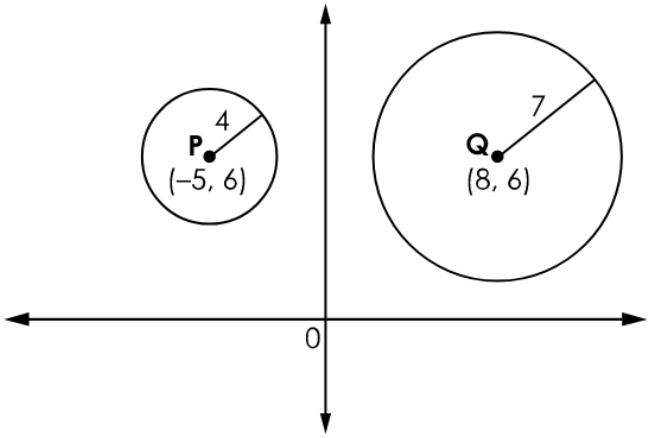
To show that circle P and circle Q are similar, show that circle P can overlay circle Q by applying a sequence of transformations—a translation by *t* units and a dilation by the scale factor *s* about the center of the translated circle P.

To get the centers of the circles to coincide, translate circle P to the right 13 units. This value can be calculated by subtracting the x-coordinates of two centers as, as $t = 8 - (-5) = 13$. So, $t = 13$ units.

The student may subtract the radius of circle Q and the radius of circle P or $7 - 4 = 3$ units instead of calculating a scale factor as $s = \frac{radius\ of\ circle\ Q}{radius\ of\ circle\ P}$ to get $s = \frac{7}{4}$.

## Sample Response: 0 points

Consider the two circles shown.




To show that circle P is similar to circle Q, circle P is translated $t$ units to the right. The image is then dilated about its center by a scale factor of $s$.

What are the values of $t$ and $s$?

$t =$ 13

$s =$ $\frac{4}{7}$

| | | |
|---|---|---|
| 1 | 2 | 3 |
| 4 | 5 | 6 |
| 7 | 8 | 9 |
| | 0 | |
| . | - | $\frac{\square}{\square}$ |

**Notes on Scoring**

This response earns no credit (0 points) because it shows the correct values for units of translation and an incorrect scale factor to prove a similarity of two circles.

To show that circle P and circle Q are similar, show that circle P can overlay circle Q by applying a sequence of transformations—a translation by *t* units and a dilation by the scale factor *s* about the center of the translated circle P.

To get the centers of the circle to coincide, translate circle P to the right 13 units. This value can be calculated by subtracting the x-coordinates of two centers as, as $t = 8 - (-5) = 13$. So, $t = 13$ units.

The student may realize that to get the circles to completely overlay, the translated image of circle P needs to be dilated by a scale factor but instead of calculating $s = \frac{radius\ of\ circle\ Q}{radius\ of\ circle\ P}$, the student reverses the ratio as $s = \frac{radius\ of\ circle\ Q}{radius\ of\ circle\ P}$ to get $s = \frac{4}{7}$.

# Geometry
# Spring 2021 Item Release

# Question 21

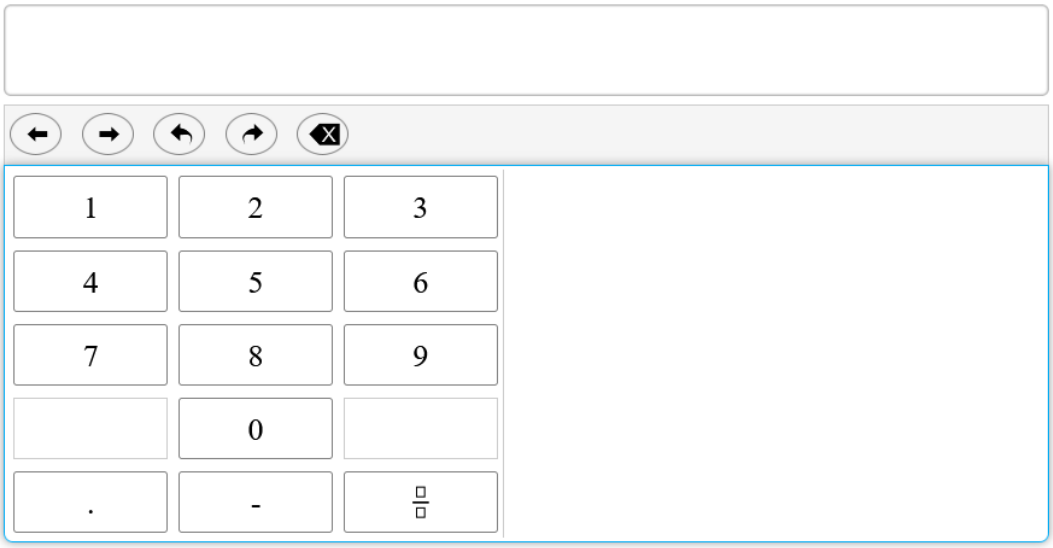
# Question and Scoring Guidelines

# Question 21

The table shows when the tickets for a concert are sold and the types of tickets that are sold.

| | Tickets Purchased on Day of Concert | Tickets Purchased in Advance | Total |
|---|---|---|---|
| **Adult** | 48 | 100 | 148 |
| **Child** | 62 | 40 | 102 |
| **Total** | 110 | 140 | 250 |

What is the probability that a randomly selected person attending the concert is an adult or has purchased the ticket in advance?



**Points Possible:** 1

**Content Cluster:** Use the rules of probability to compute probabilities of compound events in a uniform probability model.

**Content Standard:** Apply the Addition Rule, P(A or B) = P(A) + P(B) − P(A and B), and interpret the answer in terms of the model.★ *(S.CP.7)*

**Depth of Knowledge:** Level 2
i. Retrieve information from a table, graph, or figure and use it to solve a problem requiring multiple steps
j. Translate between tables, graphs, words and symbolic notation

# Scoring Guidelines

Exemplar Response

- $\frac{188}{250}$

Other Correct Responses

- any equivalent value

For this item, a full-credit response includes:

- the correct value (1 point).

# Geometry
# Spring 2021 Item Release

# Question 21

# Sample Responses

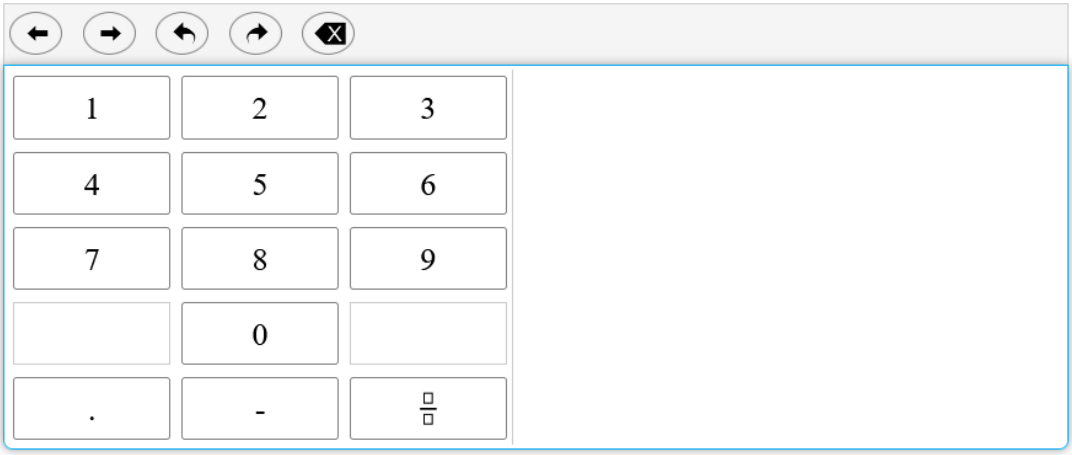
**Sample Response: 1 point**

The table shows when the tickets for a concert are sold and the types of tickets that are sold.

| | Tickets Purchased on Day of Concert | Tickets Purchased in Advance | Total |
|---|---|---|---|
| **Adult** | 48 | 100 | 148 |
| **Child** | 62 | 40 | 102 |
| **Total** | 110 | 140 | 250 |

What is the probability that a randomly selected person attending the concert is an adult or has purchased the ticket in advance?

$\frac{188}{250}$



**Notes on Scoring**

This response earns full credit (1 point) because it shows the correct probability of compound events.

The student understands that the probability that a randomly selected person attending the concert is an adult or has purchased the ticket in advance can be calculated using the Addition Rule for the probability of compound events, or $P(A\ or\ B) = P(A) + P(B) - P(A\ and\ B)$.

The probability that a person attending the concert is an adult is $P(A) = \frac{148}{250}$.

The probability that a person has purchased the ticket in advance is $P(B) = \frac{140}{250}$.

The probability that a selected person attending the concert is an adult and has purchased the ticket in advance is $P(A\ and\ B) = \frac{100}{250}$.

By substituting these values into the Addition Rule formula, complete the calculation of P(A or B) as $\frac{148}{250} + \frac{140}{250} - \frac{100}{250} = \frac{188}{250}$.

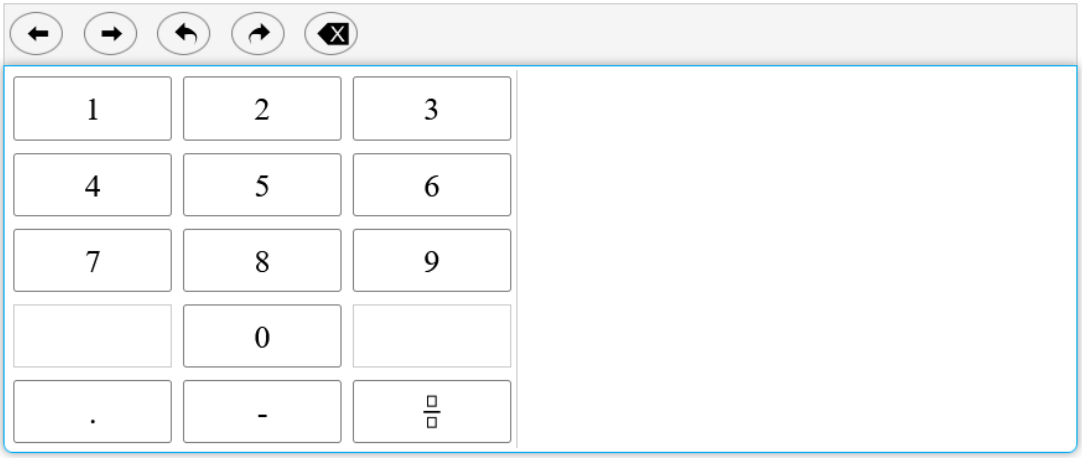
**Sample Response: 1 point**

The table shows when the tickets for a concert are sold and the types of tickets that are sold.

| | Tickets Purchased on Day of Concert | Tickets Purchased in Advance | Total |
|---|---|---|---|
| **Adult** | 48 | 100 | 148 |
| **Child** | 62 | 40 | 102 |
| **Total** | 110 | 140 | 250 |

What is the probability that a randomly selected person attending the concert is an adult or has purchased the ticket in advance?

$\frac{94}{125}$



**Notes on Scoring**

This response earns full credit (1 point) because it shows the correct probability of compound events.

The student understands that the probability that a randomly selected person attending the concert is an adult or has purchased the ticket in advance can be calculated using the Addition Rule for the probability of compound events, or $P(A\ or\ B) = P(A) + P(B) - P(A\ and\ B)$.

The probability that a person attending the concert is an adult is $P(A) = \frac{148}{250}$.

The probability that a person has purchased the ticket in advance is $P(B) = \frac{140}{250}$.

The probability that a selected person attending the concert is an adult and has purchased the ticket in advance is $P(A\ and\ B) = \frac{100}{250}$.

By substituting these values into the Addition Rule formula, complete the calculation of P(A or B) as $\frac{148}{250} + \frac{140}{250} - \frac{100}{250} = \frac{188}{250}$ or $\frac{94}{125}$.

**Sample Response: 0 points**

The table shows when the tickets for a concert are sold and the types of tickets that are sold.

| | Tickets Purchased on Day of Concert | Tickets Purchased in Advance | Total |
|---|---|---|---|
| **Adult** | 48 | 100 | 148 |
| **Child** | 62 | 40 | 102 |
| **Total** | 110 | 140 | 250 |

What is the probability that a randomly selected person attending the concert is an adult or has purchased the ticket in advance?

$\frac{148}{250}$

**Notes on Scoring**

This response earns no credit (0 points) because it shows an incorrect probability of compound events.

The student may confuse the probability that a randomly selected person attending the concert is an adult or has purchased the ticket in advance with the probability that a person attending the concert is an adult or $P(A) = \frac{148}{250}$.

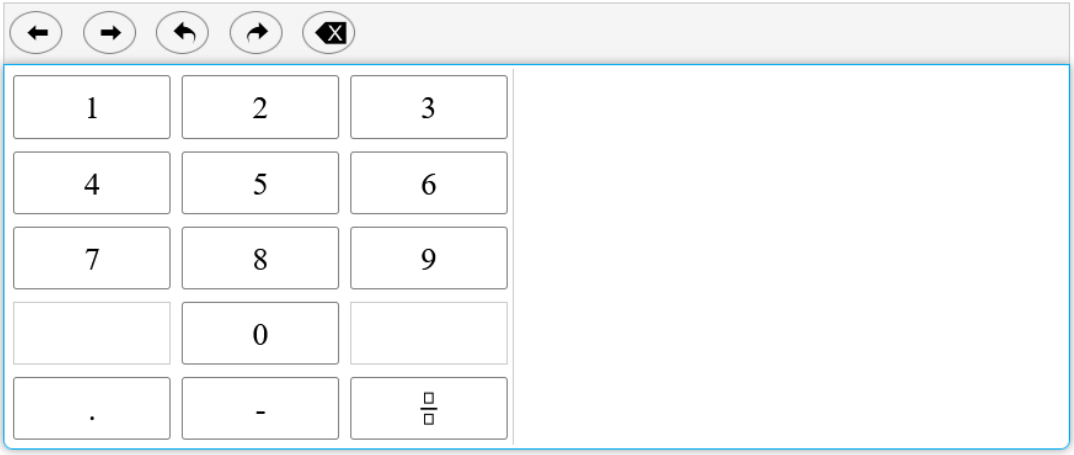
**Sample Response: 0 points**

The table shows when the tickets for a concert are sold and the types of tickets that are sold.

| | Tickets Purchased on Day of Concert | Tickets Purchased in Advance | Total |
|---|---|---|---|
| **Adult** | 48 | 100 | 148 |
| **Child** | 62 | 40 | 102 |
| **Total** | 110 | 140 | 250 |

What is the probability that a randomly selected person attending the concert is an adult or has purchased the ticket in advance?

$\frac{2}{5}$



**Notes on Scoring**

This response earns no credit (0 points) because it shows an incorrect probability of compound events.

The student may confuse the probability that a randomly selected person attending the concert is an adult or has purchased the ticket in advance with the probability that a person attending the concert is an adult who purchased the ticket in advance, $P(A) = \frac{100}{250}$ or $P(A) = \frac{2}{5}$.

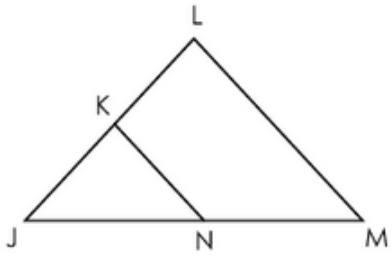
# Geometry
# Spring 2021 Item Release

# Question 22

# Question and Scoring Guidelines

# Question 22

A triangle JLM and line segment KN are given.




A proof of $\frac{JK}{JL} = \frac{JN}{JM}$ is shown.

| Statements | Reasons |
|---|---|
| $\triangle JLM$ | Given |
| ? | Given |
| $\angle JNK = \angle JML$<br>$\angle JKN = \angle JLM$ | Corresponding angles are congruent. |
| $\triangle JKN = \triangle JLM$ | Angle-angle similar triangle postulate |
| $\frac{JK}{JL} = \frac{JN}{JM}$ | Corresponding parts of similar triangles are proportional. |

Which statement must be added to the given to keep this proof valid?

(A) $\overline{JL} \perp \overline{LM}$

(B) $\overline{KN} \perp \overline{LM}$

(C) $\overline{JL} \parallel \overline{LM}$

(D) $\overline{KN} \parallel \overline{LM}$

**Points Possible:** 1

**Content Cluster:** Prove and apply theorems both formally and informally involving similarity using a variety of methods.

**Content Standard:** Prove and apply theorems about triangles. *Theorems include but are not restricted to the following: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity. (G.SRT.4)*

**Depth of Knowledge:** Level 2
g. Provide justifications for steps in a solution process
i. Retrieve information from a table, graph, or figure and use it to solve a problem requiring multiple steps
k. Make direct translations between problem situations and symbolic notation

# Scoring Guidelines

Rationale for Option A: This is incorrect. The student may assume that JL and LM are perpendicular but does not realize that lines do not need to be perpendicular to prove corresponding sides of similar triangles are proportional.

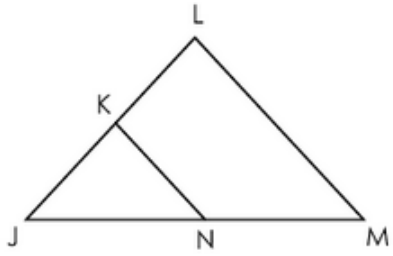
Rationale for Option B: This is incorrect. The student may assume that KN and LM are perpendicular but does not realize that lines do not need to be perpendicular to prove corresponding sides of similar triangles are proportional.

Rationale for Option C: This is incorrect. The student may realize that to prove that corresponding angles are congruent, sides KN II LM must be added to the given information but confuse the line segments and select JL instead of KN.

Rationale for Option D: **Key** – The student realizes that to prove corresponding sides of triangles are proportional, triangles need to be similar (AA criteria of similarity) and corresponding angles need to be congruent. It happens when the angles are formed by two parallel lines intercepted by a transversal line.

## Sample Response: 1 point

A triangle JLM and line segment KN are given.




A proof of $\frac{JK}{JL} = \frac{JN}{JM}$ is shown.

| Statements | Reasons |
|---|---|
| △ JLM | Given |
| ? | Given |
| ∠JNK = ∠JML<br>∠JKN = ∠JLM | Corresponding angles are congruent. |
| △ JKN = △ JLM | Angle-angle similar triangle postulate |
| $\frac{JK}{JL} = \frac{JN}{JM}$ | Corresponding parts of similar triangles are proportional. |

Which statement must be added to the given to keep this proof valid?

Ⓐ $\overline{JL} \perp \overline{LM}$

Ⓑ $\overline{KN} \perp \overline{LM}$

Ⓒ $\overline{JL} \parallel \overline{LM}$

● $\overline{KN} \parallel \overline{LM}$

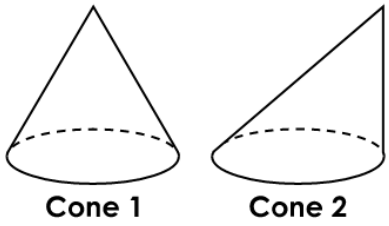
# Geometry
# Spring 2021 Item Release

# Question 23

# Question and Scoring Guidelines

# Question 23

Two cones are shown. The heights and bases of Cone 1 and Cone 2 are congruent.




In Cone 1, the vertex is directly above the center of the circle. In Cone 2, the vertex is directly above a point on the circle.

Select the phrases that correctly complete the sentences comparing the cross sections and volumes of Cone 1 and Cone 2.

The area of a cross section parallel to the base at the same height in each cone is [ ]

Therefore, the volume of Cone 1 is [ ] the volume of Cone 2.

**Drop Down Choices:**

The area of a cross section parallel to the base at the same height in each cone is [ ]

- equal for Cone 1 and Cone 2.
- larger for Cone 1 than for Cone 2.
- smaller for Cone 1 than for Cone 2.

Therefore, the volume of Cone 1 is [ ] the volume of Cone 2.

- equal to
- less than
- greater than

**Points Possible:** 1

**Content Cluster:** Explain volume formulas and use them to solve problems.

**Content Standard:** Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. *Use dissection arguments, Cavalieri's principle, and informal limit arguments. (G.GMD.1)*

**Depth of Knowledge:** Level 2
e. Compare and/or contrast figures or statements
i. Retrieve information from a table, graph, or figure and use it to solve a problem requiring multiple steps
m. Specify and explain relationships between facts, terms, properties, or operations

# Scoring Guidelines

Exemplar Response

- The area of a cross section parallel to the base at the same height in each cone is **equal for Cone 1 and Cone 2**.

  Therefore, the volume of Cone 1 is **equal to** the volume of Cone 2.

Other Correct Responses

- N/A

For this item, a full-credit response includes:

- correctly completed statements (1 point).

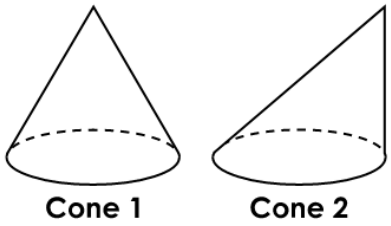
# Geometry
# Spring 2021 Item Release

# Question 23

# Sample Responses

## Sample Response: 1 point

Two cones are shown. The heights and bases of Cone 1 and Cone 2 are congruent.




In Cone 1, the vertex is directly above the center of the circle. In Cone 2, the vertex is directly above a point on the circle.

Select the phrases that correctly complete the sentences comparing the cross sections and volumes of Cone 1 and Cone 2.

The area of a cross section parallel to the base at the same height in each cone is [equal for Cone 1 and Cone 2.]

Therefore, the volume of Cone 1 is [equal to] the volume of Cone 2.

**Notes on Scoring**

This response earns full credit (1 point) because it correctly compares the cross sections and volumes of two cones.

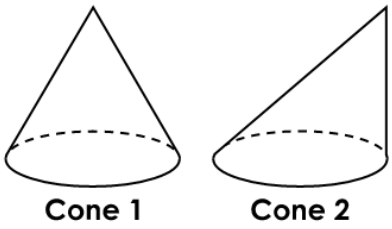
Imagine that Cone 1 has been sliced into several thin "cylindrical-looking" slices of the same thickness that are parallel to the base. The slices have then been slid to the right to form a slanted Cone 2. The student understands that the radius of each slanted slice has not changed, and that the area of its circular base ($A_{circle} = \pi r^2$) remains unchanged too. Therefore, the first correctly completed sentence is, "The area of a cross section parallel to the base at the same height in each cone is **equal for Cone 1 and Cone 2**."

If the number of slices increases, the thickness of each slice becomes extremely small, the slice becomes a "cross section," and each cone becomes a compacted stack of a very large number of extremely thin cross sections. The student understands that since Cone 1 and Cone 2 are composed of equal number of matching cross sections with equal areas, the cones have the same volumes.

The second correctly completed sentence is, "Therefore, the volume of Cone 1 is **equal to** the volume of Cone 2."

## Sample Response: 0 points

Two cones are shown. The heights and bases of Cone 1 and Cone 2 are congruent.




In Cone 1, the vertex is directly above the center of the circle. In Cone 2, the vertex is directly above a point on the circle.

Select the phrases that correctly complete the sentences comparing the cross sections and volumes of Cone 1 and Cone 2.

The area of a cross section parallel to the base at the same height in each cone is [larger for Cone 1 than for Cone 2.]

Therefore, the volume of Cone 1 is [greater than] the volume of Cone 2.

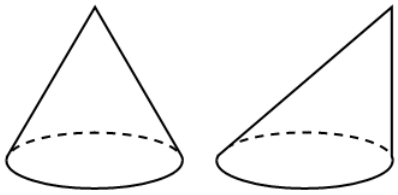
**Notes on Scoring**

This response earns no credit (0 points) because it incorrectly compares the cross sections and volumes of two cones.

The student may mistakenly believe that the area of a cross section in Cone 1 is larger than the area of a cross section in Cone 2 when they are taken at the same height. Therefore, the volume of Cone 1 is greater than the volume of Cone 2.

## Sample Response: 0 points

Two cones are shown. The heights and bases of Cone 1 and Cone 2 are congruent.




In Cone 1, the vertex is directly above the center of the circle. In Cone 2, the vertex is directly above a point on the circle.

Select the phrases that correctly complete the sentences comparing the cross sections and volumes of Cone 1 and Cone 2.

The area of a cross section parallel to the base at the same height in each cone is [smaller for Cone 1 than for Cone 2.]

Therefore, the volume of Cone 1 is [less than] the volume of Cone 2.

**Notes on Scoring**

This response earns no credit (0 points) because it incorrectly compares the cross sections and volumes of two cones.

The student may mistakenly believe that the area of a cross section in Cone 1 is smaller than the area of a cross section in Cone 2 when they are taken at the same height. Therefore, the volume of Cone 1 is less than the volume of Cone 2.

# Geometry
# Spring 2021 Item Release

# Question 25

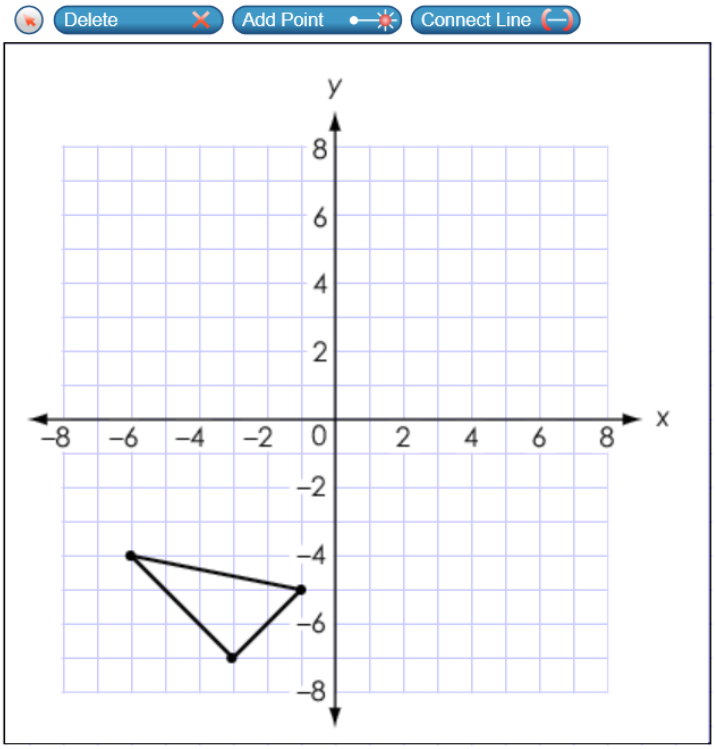
# Question and Scoring Guidelines

# Question 25

A triangle is shown.

The triangle is translated 8 units to the right, then reflected over the *x*-axis.

Use the Connect Line tool to draw the result.




**Points Possible:** 1

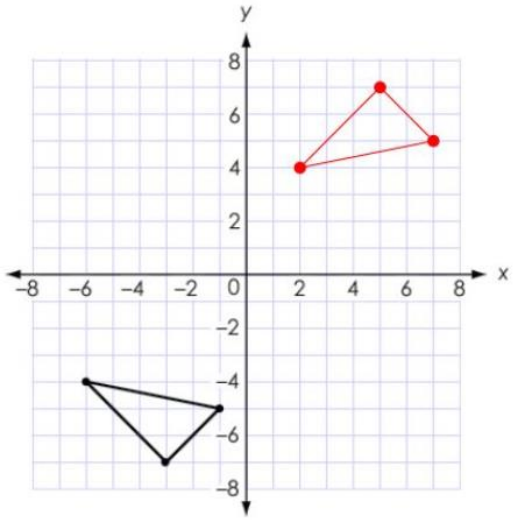
**Content Cluster:** Experiment with transformations in the plane.

**Content Standard:** Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another. *(G.CO.5)*

**Depth of Knowledge:** Level 2
i. Retrieve information from a table, graph, or figure and use it to solve a problem requiring multiple steps
l. Select a procedure according to criteria and perform it

# Scoring Guidelines

Exemplar Response




Other Correct Responses

- a graph of the final image along with the intermediate image and/or the line of reflection

For this item, a full-credit response includes:

- the correct triangle (1 point).

# Geometry
# Spring 2021 Item Release

# Question 25

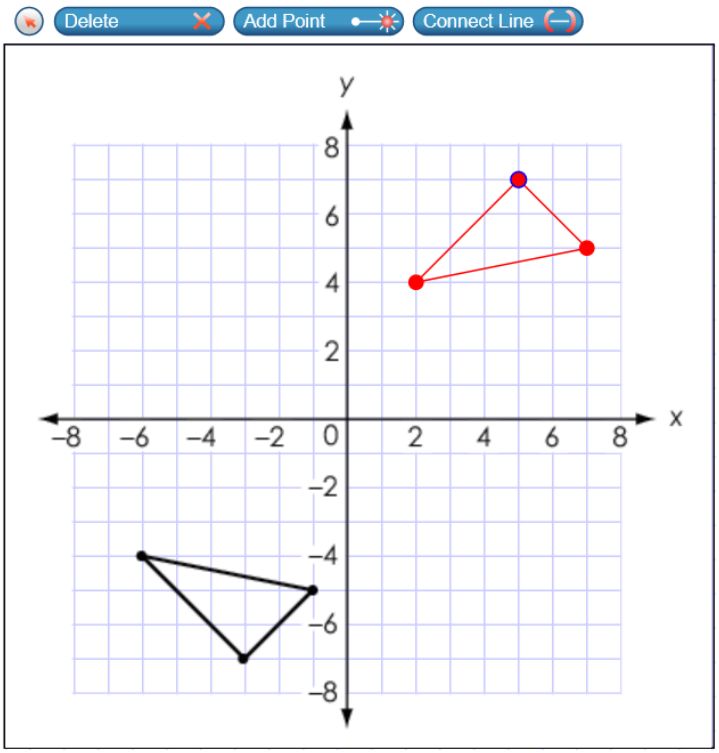
# Sample Responses

## Sample Response: 1 point

A triangle is shown.

The triangle is translated 8 units to the right, then reflected over the *x*-axis.

Use the Connect Line tool to draw the result.



**Notes on Scoring**

This response earns full credit (1 point) because it shows the correct image of the given triangle, as a result of a sequence of two transformations.

The student may construct the resulting triangle in two stages.

The translation of the triangle 8 units to the right will change the x-coordinate of the vertices as $(x, y) \rightarrow (x + 8, y)$.

The reflection of the resultant triangle over the x-axis will change the *y*-coordinate of the vertices to the opposite as $(x + 8, y) \rightarrow (x + 8, -y)$.

Therefore, the vertex (–6, –4) first moves to (2, –4) and then to (2, 4).

The vertex (–3, –7) first moves to (5, –7) and then to (5, 7).

The vertex (–1, –5) first moves to (7, –5) and then to (7, 5).

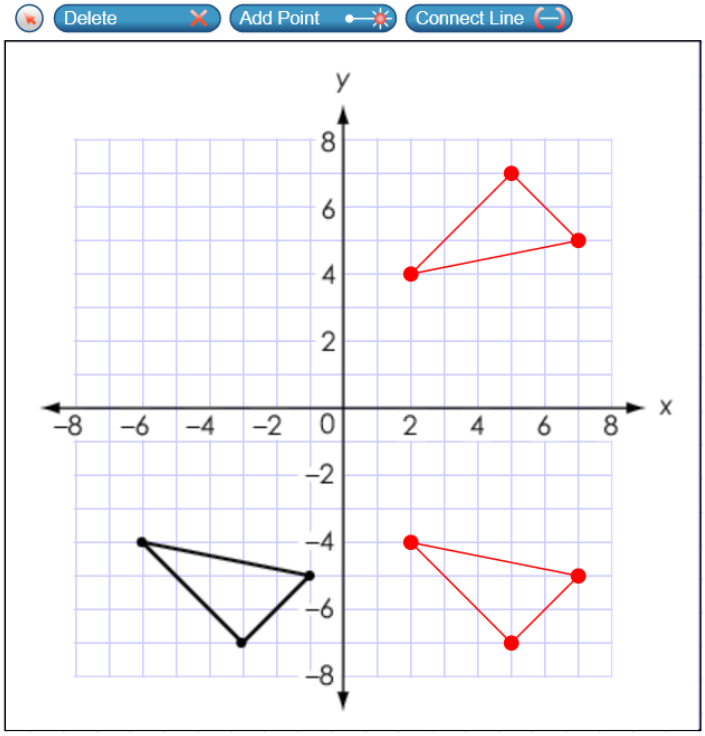
The correct final image of the given triangle has vertices at (2, 4), (5, 7) and (7, 5).

## Sample Response: 1 point

A triangle is shown.

The triangle is translated 8 units to the right, then reflected over the *x*-axis.

Use the Connect Line tool to draw the result.




**Notes on Scoring**

This response earns full credit (1 point) because it shows the correct image of the given triangle, as a result of a sequence of two transformations.

The student may construct the resulting triangle in two stages, where each stage is shown on a grid.

The translation of the triangle 8 units to the right will change the x-coordinate of the vertices as $(x, y) \rightarrow (x + 8, y)$, creating a triangle with coordinates (2, –4), (5, –7) and (7, –5).

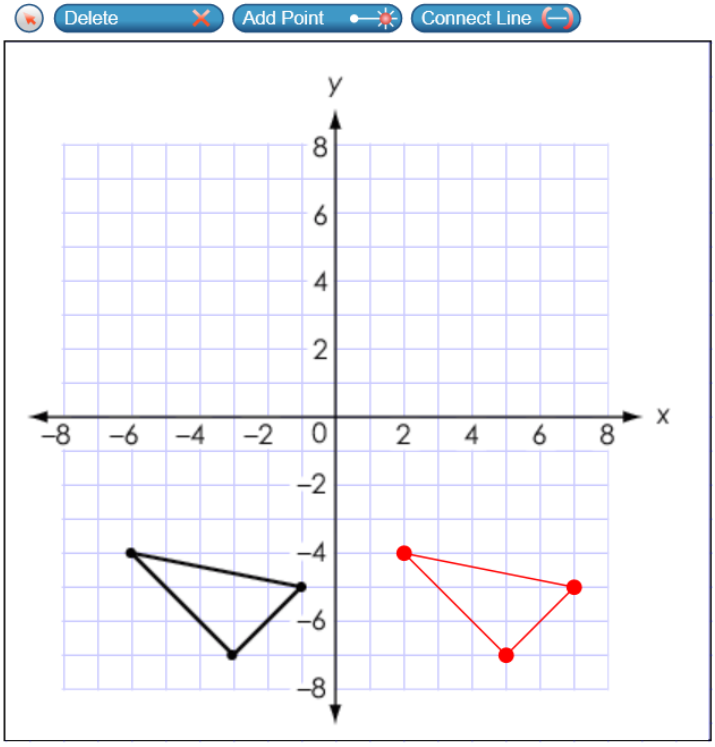
The reflection of the resultant triangle over the x-axis will change the y-coordinate of the vertices to the opposite as $(x + 8, y) \rightarrow (x + 8, -y)$, creating a final triangle with coordinates (2, 4), (5, 7) and (7, 5).

## Sample Response: 0 points

A triangle is shown.

The triangle is translated 8 units to the right, then reflected over the *x*-axis.

Use the Connect Line tool to draw the result.




**Notes on Scoring**

This response earns no credit (0 points) because it shows an incorrect image of the given triangle.

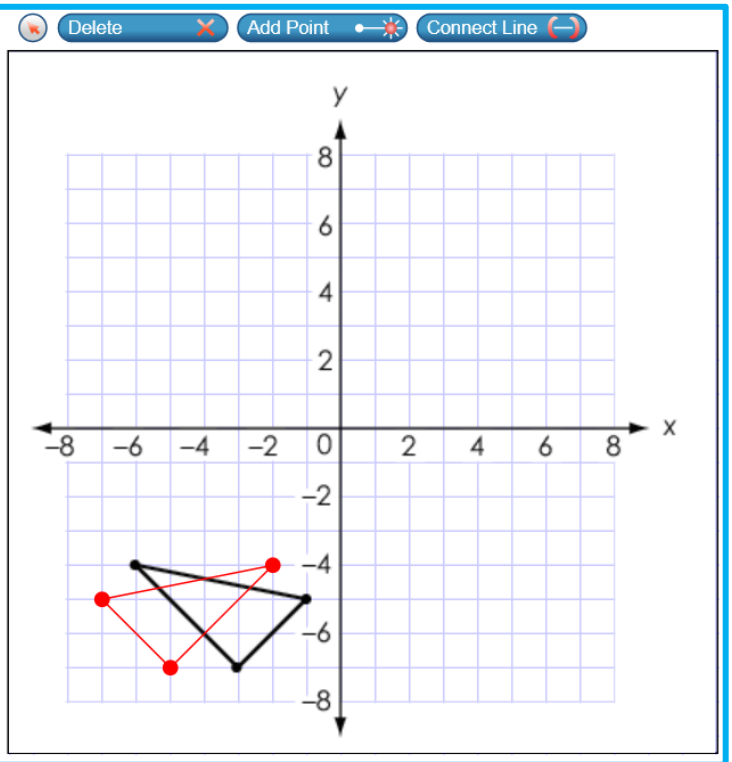
The student may neglect the reflection over the x-axis to perform only a translation of the triangle 8 units to the right, creating an image with coordinates (2, –4), (5, –7) and (7, –5).

## Sample Response: 0 points

A triangle is shown.

The triangle is translated 8 units to the right, then reflected over the *x*-axis.

Use the Connect Line tool to draw the result.




**Notes on Scoring**

This response earns no credit (0 points) because it shows an incorrect image of the given triangle.

The student may translate a given triangle 8 units to the right by changing the x-coordinate of the vertices as $(x, y) \rightarrow (x + 8, y)$, creating a triangle with coordinates (2, –4), (5, –7) and (7, –5).

Then, the student may mistakenly reflect that triangle over the *y*-axis by changing the x-coordinates to the opposites of the x-coordinates as $(x + 8, y) \rightarrow (-(x + 8), y)$, creating a final triangle with coordinates (–2, –4), (–5, –7) and (–7, –5).

# Geometry
# Spring 2021 Item Release

# Question 28

# Question and Scoring Guidelines

## Question 28

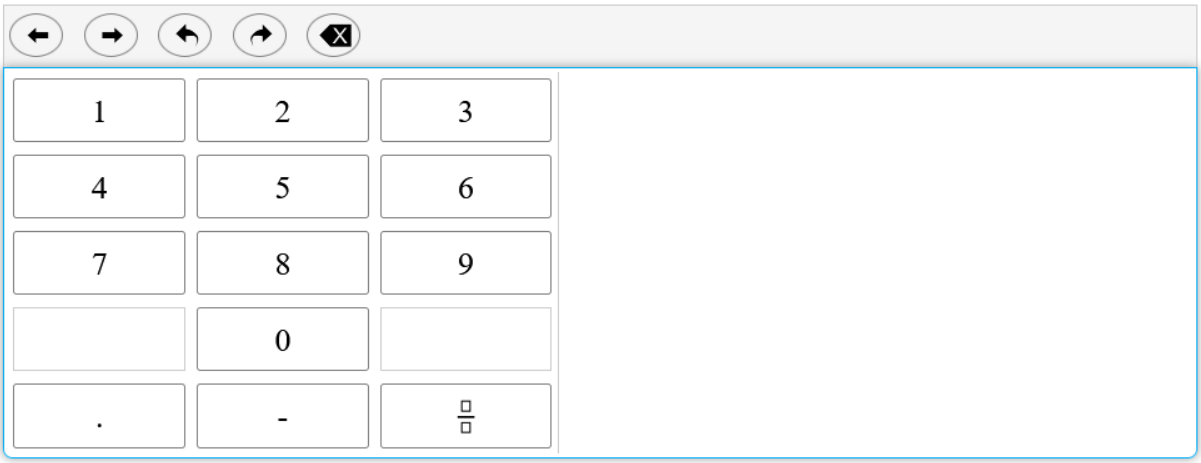
The side lengths, in units, of triangle MNO are given.

- MN = 5
- NO = 4
- OM = 2.5

Triangle MNO is dilated by a scale factor of $k$, with the center at point M, to create triangle MGH. Triangle MGH has a perimeter of 17.25 units.

What is the value of $k$?

$k =$



**Points Possible:** 1

**Content Cluster:** Understand similarity in terms of similarity transformations.

**Content Standard:** Verify experimentally the properties of dilations[G] given by a center and a scale factor.
*b.* The dilation of a line segment is longer or shorter in the ratio given by the scale factor. *(G.SRT.1)*

**Depth of Knowledge:** Level 2
d. Solve a routine problem requiring multiple steps/decision points, or the application of multiple concepts

# Scoring Guidelines

Exemplar Response

- $k = \frac{3}{2}$

Other Correct Responses

- any equivalent value

For this item, a full-credit response includes:

- a correct value (1 point).

# Geometry
# Spring 2021 Item Release

# Question 28

# Sample Responses

## Sample Response: 1 point

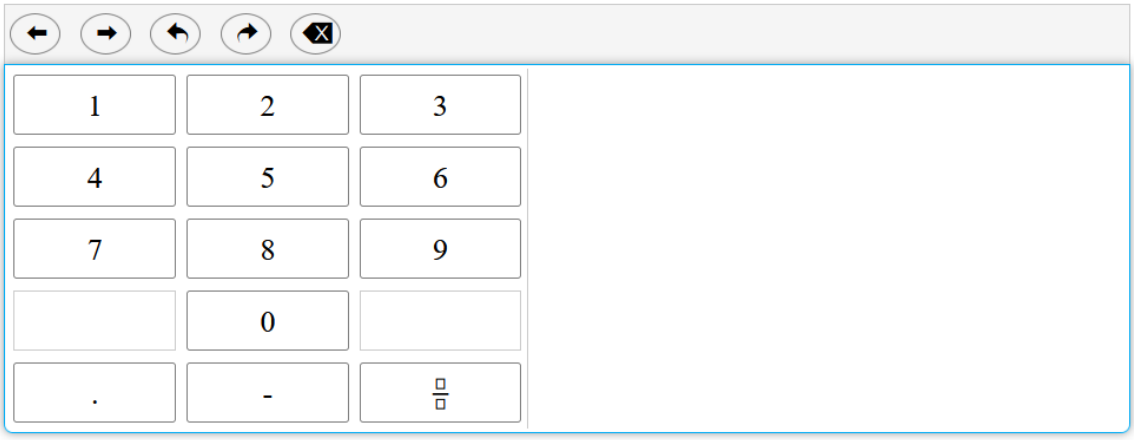
The side lengths, in units, of triangle MNO are given.

- MN = 5
- NO = 4
- OM = 2.5

Triangle MNO is dilated by a scale factor of $k$, with the center at point M, to create triangle MGH. Triangle MGH has a perimeter of 17.25 units.

What is the value of $k$?

$k = \frac{3}{2}$



**Notes on Scoring**

This response earns full credit (1 point) because it shows the correct value of a scale factor for a dilation.

When a triangle is dilated by a scale factor, *k*, all side lengths and perimeter are getting enlarged by the scale factor, *k*, as $P_{MGH} = k \bullet P_{MNO}$ regardless of the dilation center.

The student calculates the perimeter of triangle MNO as the sum of all side lengths, or $5 + 4 + 2.5 = 11.5$ units. The perimeter of the triangle MGH is 17.25 units.

So, the scale factor is $k = \frac{Perimeter\ of\ MGH}{Perimeter\ of\ MNO} = \frac{17.25}{11.5}$ or $k = \frac{3}{2}$.

## Sample Response: 1 point

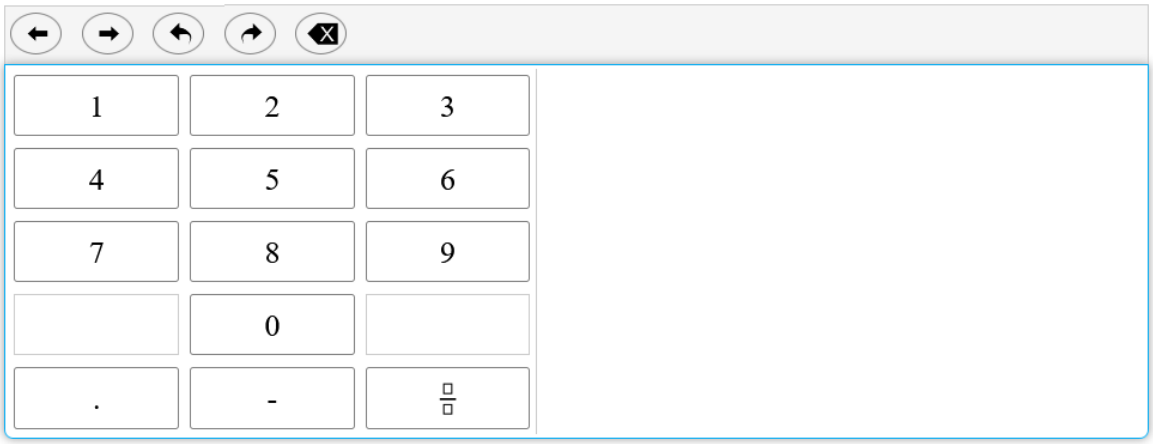
The side lengths, in units, of triangle MNO are given.

- MN = 5
- NO = 4
- OM = 2.5

Triangle MNO is dilated by a scale factor of $k$, with the center at point M, to create triangle MGH. Triangle MGH has a perimeter of 17.25 units.

What is the value of $k$?

$k =$ 1.5



**Notes on Scoring**

This response earns full credit (1 point) because it shows the correct value of a scale factor for a dilation.

When a triangle is dilated by a scale factor, *k*, all side lengths and perimeter are getting enlarged by the scale factor, *k*, as $P_{MGH} = k \bullet P_{MNO}$ regardless of the dilation center.

The student calculates the perimeter of triangle MNO is the sum of all side lengths, or $5 + 4 + 2.5 = 11.5$ units. The perimeter of the triangle MGH is 17.25 units.

So, the scale factor is $k = \frac{Perimeter\ of\ MGH}{Perimeter\ of\ MNO} = \frac{17.25}{11.5}$, $k = \frac{3}{2}$ or $k = 1.5$.

## Sample Response: 0 points

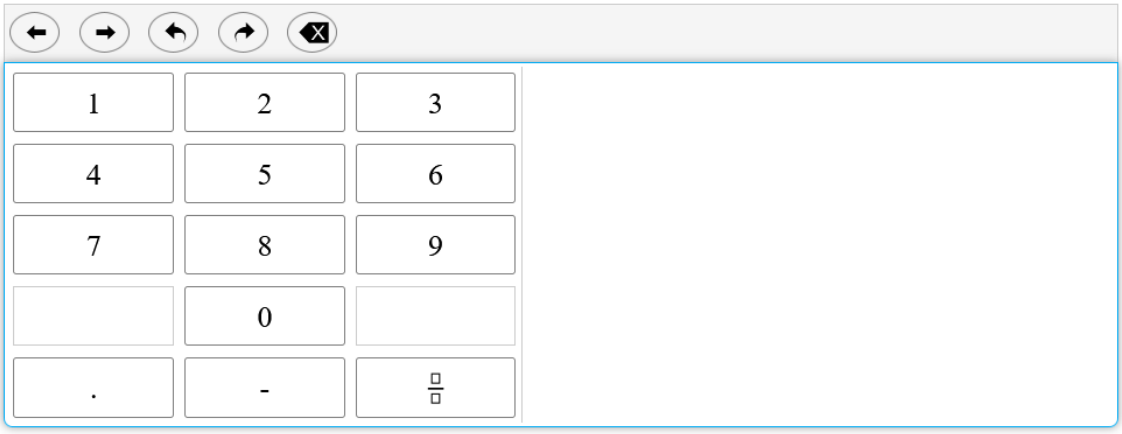
The side lengths, in units, of triangle MNO are given.

- MN = 5
- NO = 4
- OM = 2.5

Triangle MNO is dilated by a scale factor of $k$, with the center at point M, to create triangle MGH. Triangle MGH has a perimeter of 17.25 units.

What is the value of $k$?

$k = \frac{2}{3}$



**Notes on Scoring**

This response earns no credit (0 points) because it shows an incorrect value of a scale factor for a dilation.

When a triangle is dilated by a scale factor, *k*, all side lengths and perimeter are getting enlarged by this scale factor, as $P_{MGH} = k \bullet P_{MNO}$ regardless of the dilation center.

The student may confuse the image, triangle MGH, and preimage, the original triangle MNO, to incorrectly conclude that $P_{MNO} = k \bullet P_{MGH}$.

Then, the student may calculate the perimeter of triangle MNO as the sum of all side lengths or $5 + 4 + 2.5 = 11.5$ units and the perimeter of triangle MGH as 17.25 units. Next, the student may incorrectly create the ratio for the scale factor as $k = \frac{Perimeter\ of\ MNO}{Perimeter\ of\ MGH} = \frac{11.5}{17.25}$ or $k = \frac{2}{3}$.

**Sample Response: 0 points**

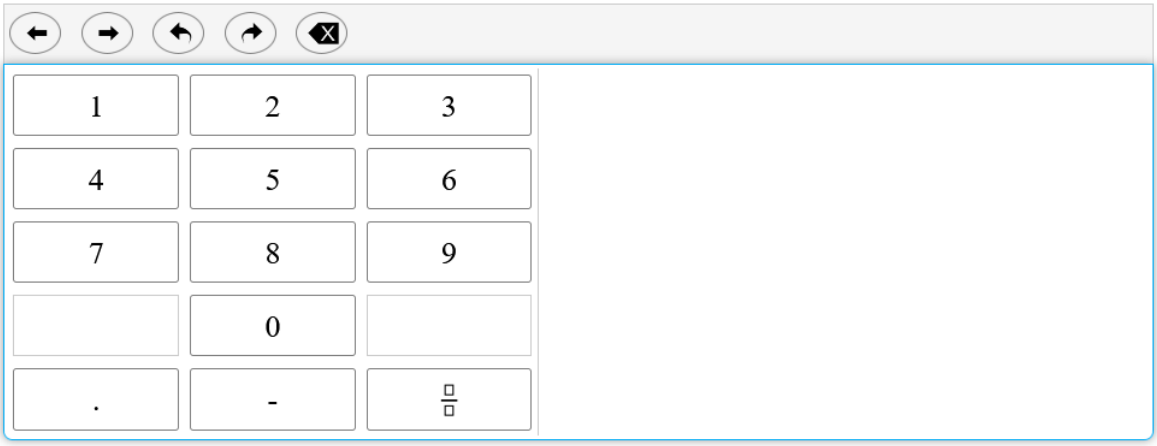
The side lengths, in units, of triangle MNO are given.

- MN = 5
- NO = 4
- OM = 2.5

Triangle MNO is dilated by a scale factor of $k$, with the center at point M, to create triangle MGH. Triangle MGH has a perimeter of 17.25 units.

What is the value of $k$?

$k =$ 0.667



**Notes on Scoring**

This response earns no credit (0 points) because it shows an incorrect value of a scale factor for a dilation.

When a triangle is dilated by a scale factor, *k*, all side lengths and perimeter are getting enlarged by the scale factor, *k*, as $P_{MGH} = k \bullet P_{MNO}$ regardless of the dilation center.

The student may confuse the image, triangle MGH, and preimage, the original triangle MNO, to incorrectly conclude that $P_{MNO} = k \bullet P_{MGH}$.

Then, the student may calculate the perimeter of triangle MNO as the sum of all side lengths or $5 + 4 + 2.5 = 11.5$ units and the perimeter of triangle MGH as 17.25 units. Next, the student may incorrectly create the ratio for the scale factor as $k = \frac{Perimeter\ of\ MNO}{Perimeter\ of\ MGH} = \frac{11.5}{17.25}$, $k = \frac{2}{3}$ or $k = 0.667$, when rounded to the nearest thousandths place.

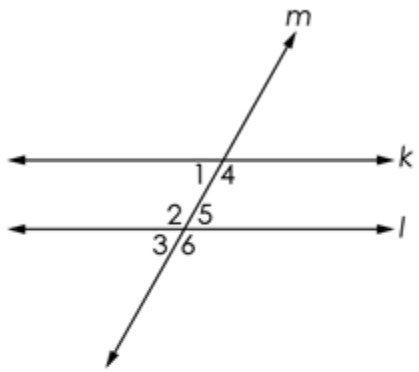
# Geometry
# Spring 2021 Item Release

# Question 37

# Question and Scoring Guidelines

# Question 37

A diagram is shown, where $k \parallel l$ and $m$ is a transversal.




Move statements and reasons to the table to prove that $\angle 1 \cong \angle 5$.

| Statements | Reasons |
|---|---|
| 1. $k \parallel l$ | 1. Given |
| 2. | 2. Corresponding angles are congruent. |
| 3. | 3. |
| 4. $\angle 1 \cong \angle 5$ | 4. |

$\angle 1 \cong \angle 2$ $\angle 1 \cong \angle 3$ $\angle 1 \cong \angle 4$ $\angle 2 \cong \angle 3$
$\angle 2 \cong \angle 4$ $\angle 2 \cong \angle 5$ $\angle 2 \cong \angle 6$ $\angle 3 \cong \angle 4$
$\angle 3 \cong \angle 5$ $\angle 4 \cong \angle 5$ $\angle 4 \cong \angle 6$
Transitive property Symmetric property
Vertical angles are congruent.
Straight angles form a linear pair.
Corresponding angles are congruent.
Alternate exterior angles are congruent.

**Points Possible:** 1

**Content Cluster:** Prove geometric theorems both formally and informally using a variety of methods.

**Content Standard:** Prove and apply theorems about lines and angles. *Theorems include but are not restricted to the following: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints. (G.CO.9)*

**Depth of Knowledge:** Level 3
a. Interpret information from a complex graph
e. Use concepts to solve non-routine problems
k. Solve a multiple-step problem and provide support with a mathematical explanation that justifies the answer

# Scoring Guidelines

For full credit (1 point), the student's response satisfies the bullet below.

- The student completes the proof that alternate interior angles are congruent using corresponding angles, vertical angles and the transitive property, providing evidence of the ability to prove theorems about lines and angles.

Exemplar Response

| Statements | Reasons |
|---|---|
| 1. $k \parallel l$ | 1. Given |
| 2. $\angle 1 \cong \angle 3$ | 2. Corresponding angles are congruent. |
| 3. $\angle 3 \cong \angle 5$ | 3. Vertical angles are congruent. |
| 4. $\angle 1 \cong \angle 5$ | 4. Transitive property |

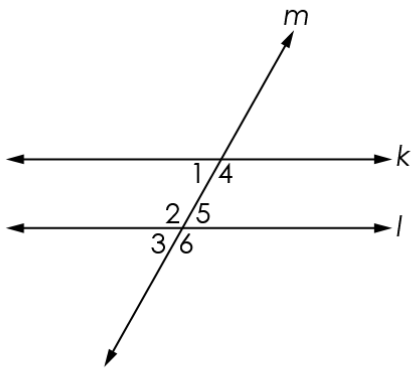
# Geometry
# Spring 2021 Item Release

# Question 37

# Sample Responses

## Sample Response: 1 point

A diagram is shown, where $k \parallel l$ and $m$ is a transversal.




Move statements and reasons to the table to prove that $\angle 1 \cong \angle 5$.

| Statements | Reasons |
|---|---|
| 1. $k \parallel l$ | 1. Given |
| 2. $\angle 1 \cong \angle 3$ | 2. Corresponding angles are congruent. |
| 3. $\angle 3 \cong \angle 5$ | 3. Vertical angles are congruent. |
| 4. $\angle 1 \cong \angle 5$ | 4. Transitive property |

$\angle 1 \cong \angle 2$ $\angle 1 \cong \angle 3$ $\angle 1 \cong \angle 4$ $\angle 2 \cong \angle 3$

$\angle 2 \cong \angle 4$ $\angle 2 \cong \angle 5$ $\angle 2 \cong \angle 6$ $\angle 3 \cong \angle 4$

$\angle 3 \cong \angle 5$ $\angle 4 \cong \angle 5$ $\angle 4 \cong \angle 6$

Transitive property Symmetric property

Vertical angles are congruent.

Straight angles form a linear pair.

Corresponding angles are congruent.

Alternate exterior angles are congruent.

**Notes on Scoring**

This response earns full credit (1 point) because it correctly completes the proof to show the two angles are congruent.

To complete a mathematics proof, a correctly chosen piece of missing information should show where each statement is logically connected with the previous statement, which then allows for the next correct statement.

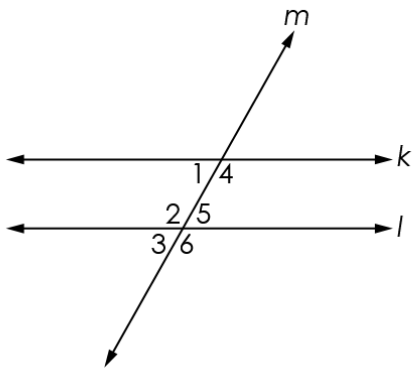
Since lines *k* and *l* are given to be parallel and *m* is a transversal, the lines form several pairs of congruent angles. So, to match Reason 2 with the correct statement option, the student selects $\angle 1 \cong \angle 3$ for Statement 2.

Lines *k*, *l* and *m* form four pairs of vertical angles. Within each pair, vertical angles are congruent. Therefore, selecting $\angle 3 \cong \angle 5$ for Statement 3 and "Vertical angles are congruent" for Reason 3 is the only option to connect Statements 2 and 4 in one step.

The use of the "Transitive Property" (Reason 4) completes the proof because Statement 4 shows $\angle 1 \cong \angle 5$ which follows from the two previous statements.

## Sample Response: 0 points

A diagram is shown, where $k \parallel l$ and $m$ is a transversal.




Move statements and reasons to the table to prove that $\angle 1 \cong \angle 5$.

| Statements | Reasons |
|---|---|
| 1. $k \parallel l$ | 1. Given |
| 2. $\angle 1 \cong \angle 3$ | 2. Corresponding angles are congruent. |
| 3. $\angle 3 \cong \angle 5$ | 3. Vertical angles are congruent. |
| 4. $\angle 1 \cong \angle 5$ | 4. Symmetric property |

$\angle 1 \cong \angle 2$ $\angle 1 \cong \angle 3$ $\angle 1 \cong \angle 4$ $\angle 2 \cong \angle 3$
$\angle 2 \cong \angle 4$ $\angle 2 \cong \angle 5$ $\angle 2 \cong \angle 6$ $\angle 3 \cong \angle 4$
$\angle 3 \cong \angle 5$ $\angle 4 \cong \angle 5$ $\angle 4 \cong \angle 6$

Transitive property
Symmetric property
Vertical angles are congruent.
Straight angles form a linear pair.
Corresponding angles are congruent.
Alternate exterior angles are congruent.

**Notes on Scoring**

This response earns no credit (0 points) because it shows an incorrect placement of one reason.

To complete a mathematics proof, a correctly chosen piece of missing information should show where each statement is logically connected with the previous statement, which then allows for the next correct statement.

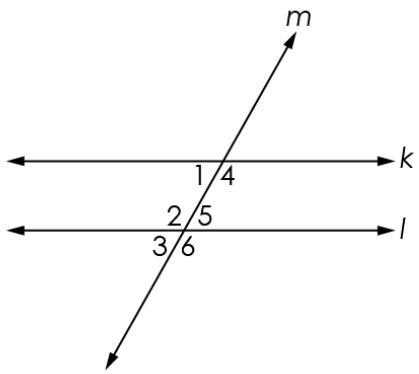
Since lines *k* and *l* are given to be parallel and *m* is a transversal, the lines form several pairs of congruent angles. So, to match Reason 2 with the correct statement option, the student selects $\angle 1 \cong \angle 3$ for Statement 2.

Lines *k*, *l* and *m* form four pairs of vertical angles. Within each pair, vertical angles are congruent. Therefore, selecting $\angle 3 \cong \angle 5$ for Statement 3 and "Vertical angles are congruent" for Reason 3 is the only option to connect Statements 2 and 4 in one step.

The use of the "Transitive property" (Reason 4) should complete the proof but the student incorrectly selects the "Symmetric property" instead.

## Sample Response: 0 points

A diagram is shown, where $k \parallel l$ and $m$ is a transversal.




Move statements and reasons to the table to prove that $\angle 1 \cong \angle 5$.

| Statements | Reasons |
|---|---|
| 1. $k \parallel l$ | 1. Given |
| 2. $\angle 3 \cong \angle 5$ | 2. Corresponding angles are congruent. |
| 3. $\angle 1 \cong \angle 3$ | 3. Vertical angles are congruent. |
| 4. $\angle 1 \cong \angle 5$ | 4. Symmetric property |

$\angle 1 \cong \angle 2$ $\angle 1 \cong \angle 3$ $\angle 1 \cong \angle 4$ $\angle 2 \cong \angle 3$

$\angle 2 \cong \angle 4$ $\angle 2 \cong \angle 5$ $\angle 2 \cong \angle 6$ $\angle 3 \cong \angle 4$

$\angle 3 \cong \angle 5$ $\angle 4 \cong \angle 5$ $\angle 4 \cong \angle 6$

Transitive property Symmetric property

Vertical angles are congruent.

Straight angles form a linear pair.

Corresponding angles are congruent.

Alternate exterior angles are congruent.

**Notes on Scoring**

This response earns no credit (0 points) because it shows an incorrect placement of three answer options—statements and reasons.

To complete a mathematics proof, a correctly chosen piece of missing information should show where each statement is logically connected with the previous statement, which then allows for the next correct statement.

Since lines *k* and *l* are given to be parallel and *m* is a transversal, the lines form several pairs of congruent angles. So, the student should select $\angle 1 \cong \angle 3$ for Statement 2 to match the given Reason 2 and select $\angle 3 \cong \angle 5$ for Statement 3 and "Vertical angles are congruent" for Reason 3. However, the student may reverse the order of Reason 2 and Reason 3 to create incorrect logic in the proof and selects a wrong Reason 4 to complete the proof.

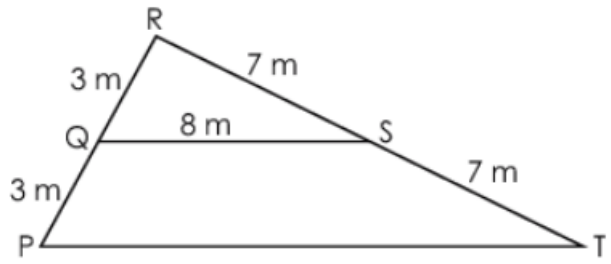
# Geometry
# Spring 2021 Item Release

# Question 40

# Question and Scoring Guidelines

# Question 40

A figure is shown.




What is the perimeter, in meters (m), of △PRT?

$m$

**Points Possible:** 1

**Content Cluster:** Prove geometric theorems both formally and informally using a variety of methods.

**Content Standard:** Prove and apply theorems about triangles. *Theorems include but are not restricted to the following: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point. (G.CO.10)*

**Depth of Knowledge:** Level 2
i. Retrieve information from a table, graph, or figure and use it to solve a problem requiring multiple steps
l. Select a procedure according to criteria and perform it

# Scoring Guidelines

Exemplar Response

- 36 m

Other Correct Responses

- any equivalent value

For this item, a full-credit response includes:

- a correct value (1 point).

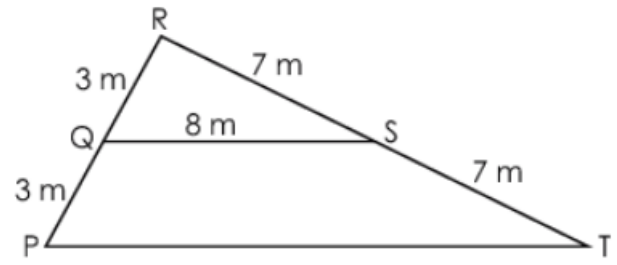
# Geometry
# Spring 2021 Item Release

# Question 40

# Sample Responses

**Sample Response: 1 point**

A figure is shown.




What is the perimeter, in meters (m), of △PRT?

36 *m*

**Notes on Scoring**

This response earns full credit (1 point) because it shows a correct value for the perimeter of the triangle.

The student calculates the perimeter of triangle PRT (or $P_{PRT}$) as PR + RT + PT or $P_{PRT} = (3 + 3) + (7 + 7) + PT$ and $P_{PRT} = 6 + 14 +$ PT.

Since PQ = QR = 3 m and RS = ST = 7 m, points Q and S are midpoints of sides PR and RT.

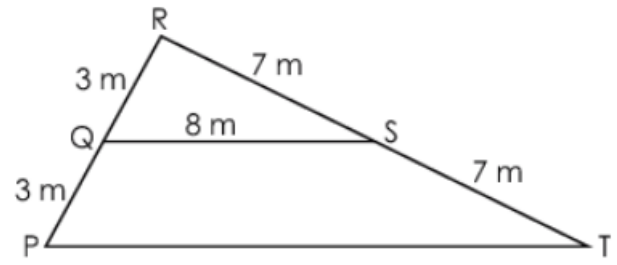
According to the Triangle Midline Theorem, a line segment QS joining the midpoints of two sides of the triangle PRT is parallel to the third side, PT, and has a length that is one-half the length of the third side, PT.

Therefore, QS = $\frac{1}{2}$ • PT and PT = 2 • QS or PT = 2 • 8 = 16 m.

Substituting PT = 16 in the formula $P_{PRT}$ = 6 + 14 + PT, the perimeter of the triangle PRT is $P_{PRT}$ = 6 + 14 + 16 = 36 m.

**Sample Response: 1 point**

A figure is shown.




What is the perimeter, in meters (m), of △PRT?

36.00 *m*

**Notes on Scoring**

This response earns full credit (1 point) because it shows a correct value for the perimeter of the triangle.

The student calculates the perimeter of triangle PRT (or $P_{PRT}$) as PR + RT + PT or $P_{PRT}$ = (3 + 3) + (7 + 7) + PT and $P_{PRT}$ = 6 + 14 + PT.

Since PQ = QR = 3 m and RS = ST = 7 m, points Q and S are midpoints of sides PR and RT.

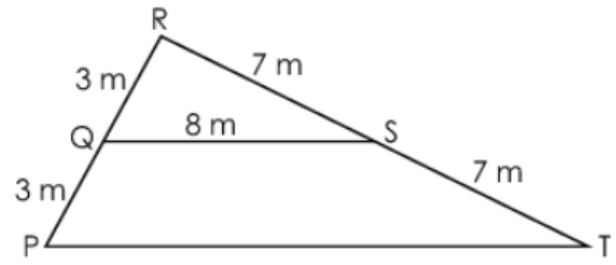
According to the Triangle Midline Theorem, a line segment QS joining the midpoints of two sides of the triangle PRT is parallel to the third side, PT, and has a length that is one-half the length of the third side, PT.

Therefore, QS = $\frac{1}{2}$ • PT and PT = 2 • QS or PT = 2 • 8 = 16 m.

Substituting PT = 16 in the formula $P_{PRT}$ = 6 + 14 + PT, the perimeter of the triangle PRT is $P_{PRT}$ = 6 + 14 + 16 = 36 m or 36.00 m.

## Sample Response: 0 points

A figure is shown.




What is the perimeter, in meters (m), of △PRT?

20 *m*

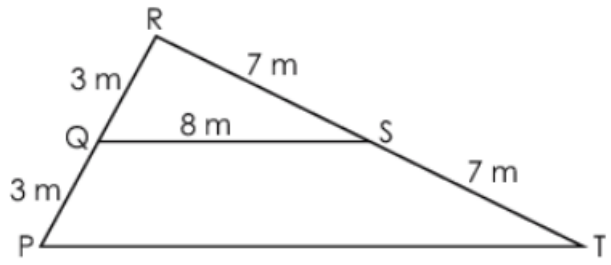

**Notes on Scoring**

This response earns no credit (0 points) because it shows an incorrect value for the perimeter of the triangle.

Since PQ = QR = 3m and RS = ST = 7m, points Q and S are midpoints of sides PR and RT. So, $PR = 6$ m and RT = 14 m. The perimeter of triangle PRT is PR + RT + PT. The student may neglect the missing side of triangle PRT and only add PR and RT or 6 + 14 = 20 m in an attempt to calculate the perimeter.

## Sample Response: 0 points

A figure is shown.




What is the perimeter, in meters (m), of △PRT?

28 *m*


**Notes on Scoring**

This response earns no credit (0 points) because it shows an incorrect value for the perimeter of the triangle.

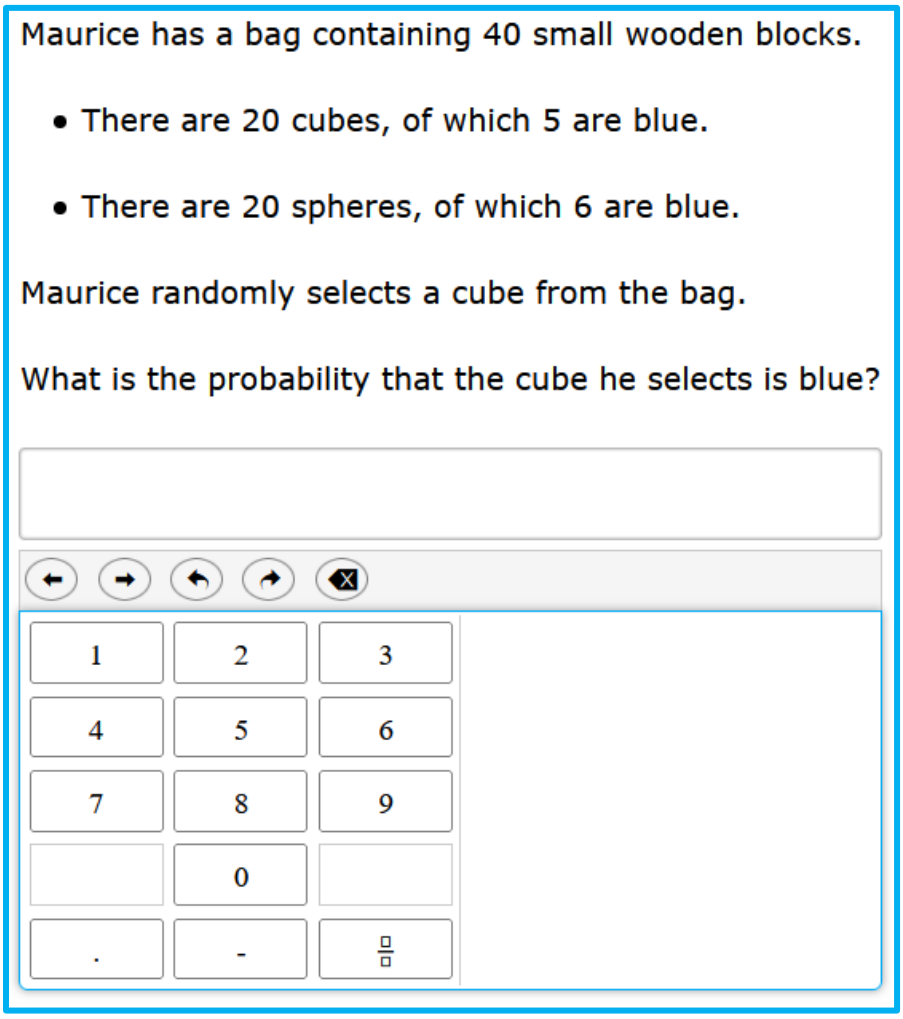
The student may notice that side lengths of the smaller triangle are two times to create the side lengths of the longer triangle. However, the student does not recognize that the side PT of the triangle PRT is also twice as long and incorrectly concludes that side PT = 8 m. Then, the student may add all three side lengths to get a perimeter 6 + 8 + 14 = 28 m.

# Geometry
# Spring 2021 Item Release

# Question 41

# Question and Scoring Guidelines

# Question 41




**Points Possible:** 1

**Content Cluster:** Use the rules of probability to compute probabilities of compound events in a uniform probability model.

**Content Standard:** Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and interpret the answer in terms of the model.★ *(S.CP.6)*

**Depth of Knowledge:** Level 2
k. Make direct translations between problem situations and symbolic notation
l. Select a procedure according to criteria and perform it

# Scoring Guidelines

For full credit (1 point), the student's response satisfies the bullet below.

- The student enters a probability of 0.25, providing evidence of the ability to find the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and interpret the answer in terms of the model.

Exemplar Response

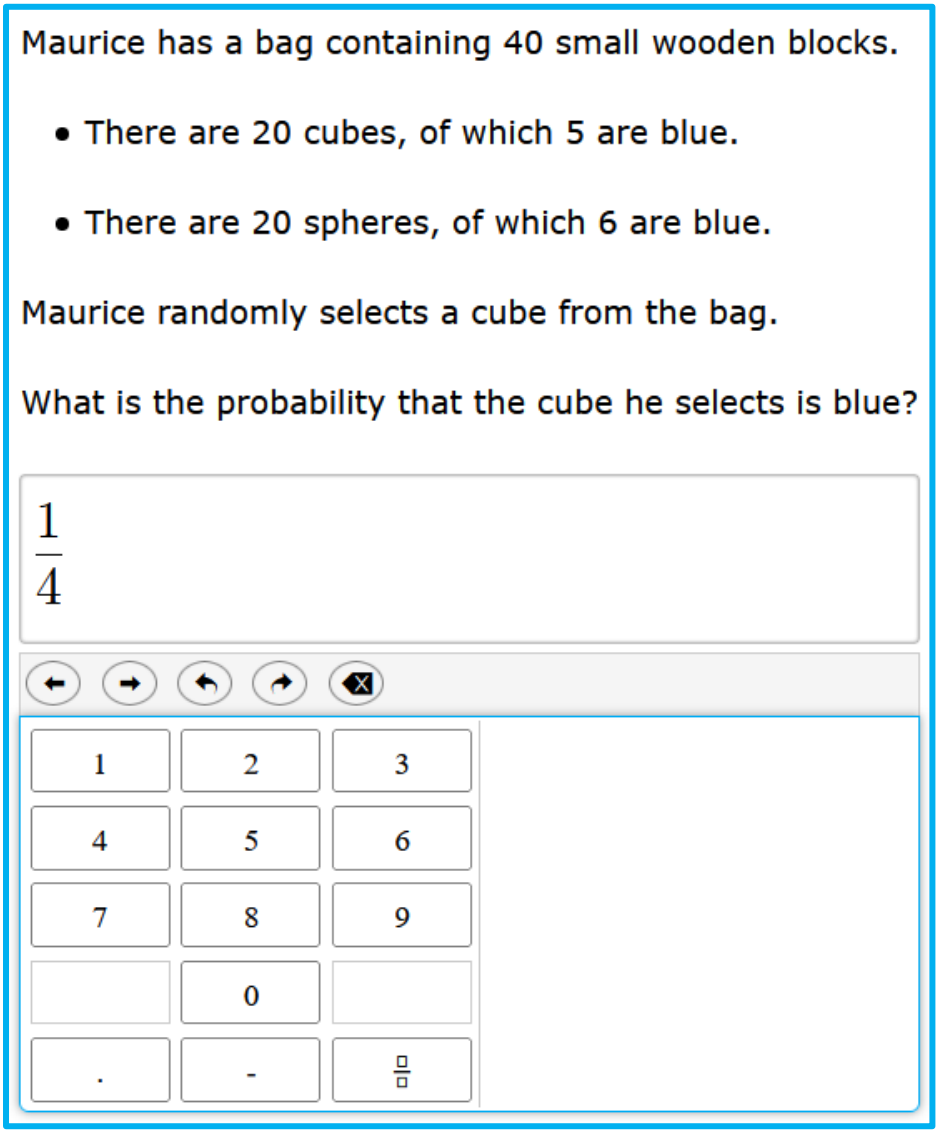
- 0.25, or any equivalent value

# Geometry
# Spring 2021 Item Release

# Question 41

# Sample Responses

**Sample Response: 1 point**



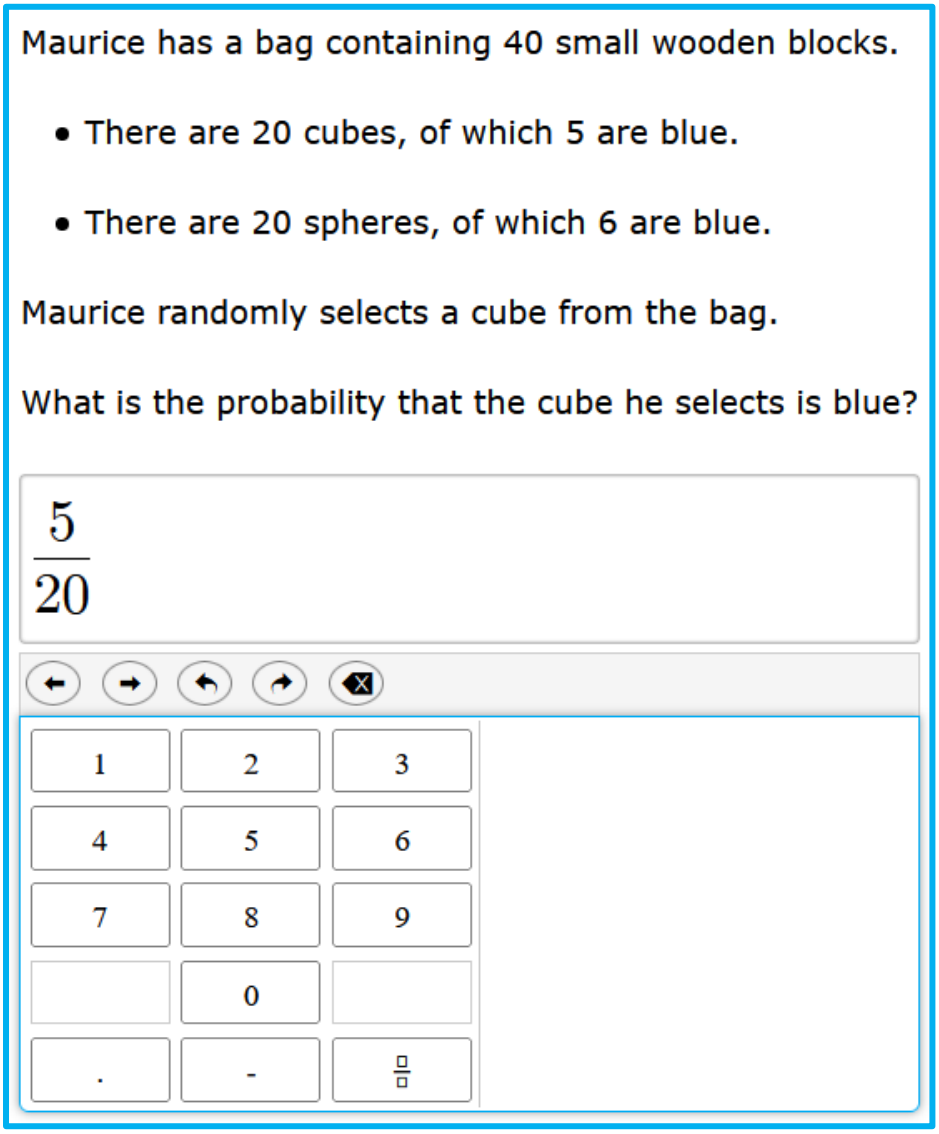
**Notes on Scoring**

This response earns full credit (1 point) because it shows the correct conditional probability.

To calculate the correct probability, it may be helpful to reword the question as, "What is the probability that the randomly selected wooden block is blue, given that the block is a cube?"

The student recognizes the presence of the wooden sphere blocks in the bag does not impact the probability of randomly drawing the blue block from the wooden cubes. It means that if 5 out of 20 wooden cubes in the bag are blue, then the probability of pulling one blue wooden cube is $\frac{5}{20}$ or $\frac{1}{4}$, when reduced to the lowest form.

**Sample Response: 1 point**



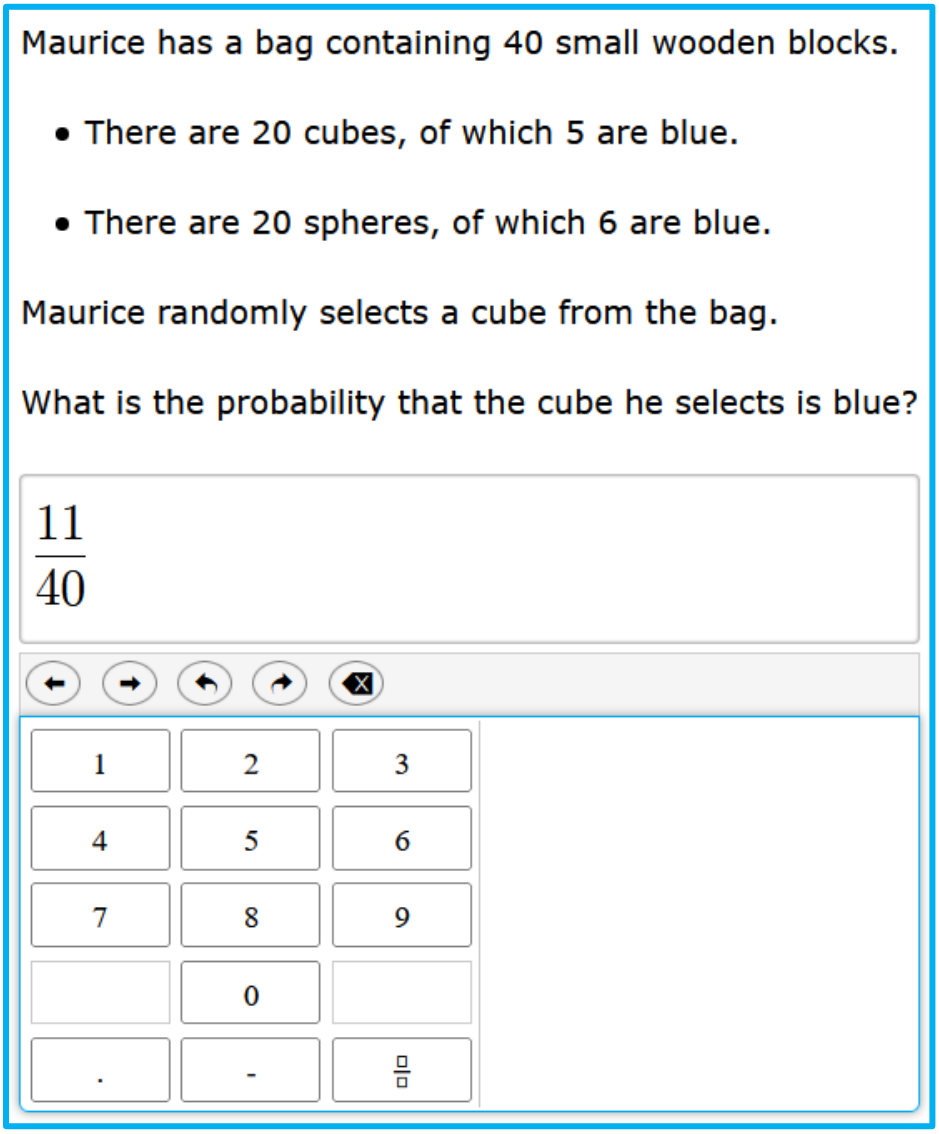
**Notes on Scoring**

This response earns full credit (1 point) because it shows the correct conditional probability.

To calculate the correct probability, it may be helpful to reword the question as, "What is the probability that the randomly selected wooden block is blue, given that the block is a cube?"

The student recognizes the presence of the wooden sphere blocks in the bag does not impact the probability of purposely drawing the blue block from the wooden cubes. It means that if 5 out of 20 wooden cubes in the bag are blue, then the probability of pulling one blue wooden cube is $\frac{5}{20}$.

## Sample Response: 0 points

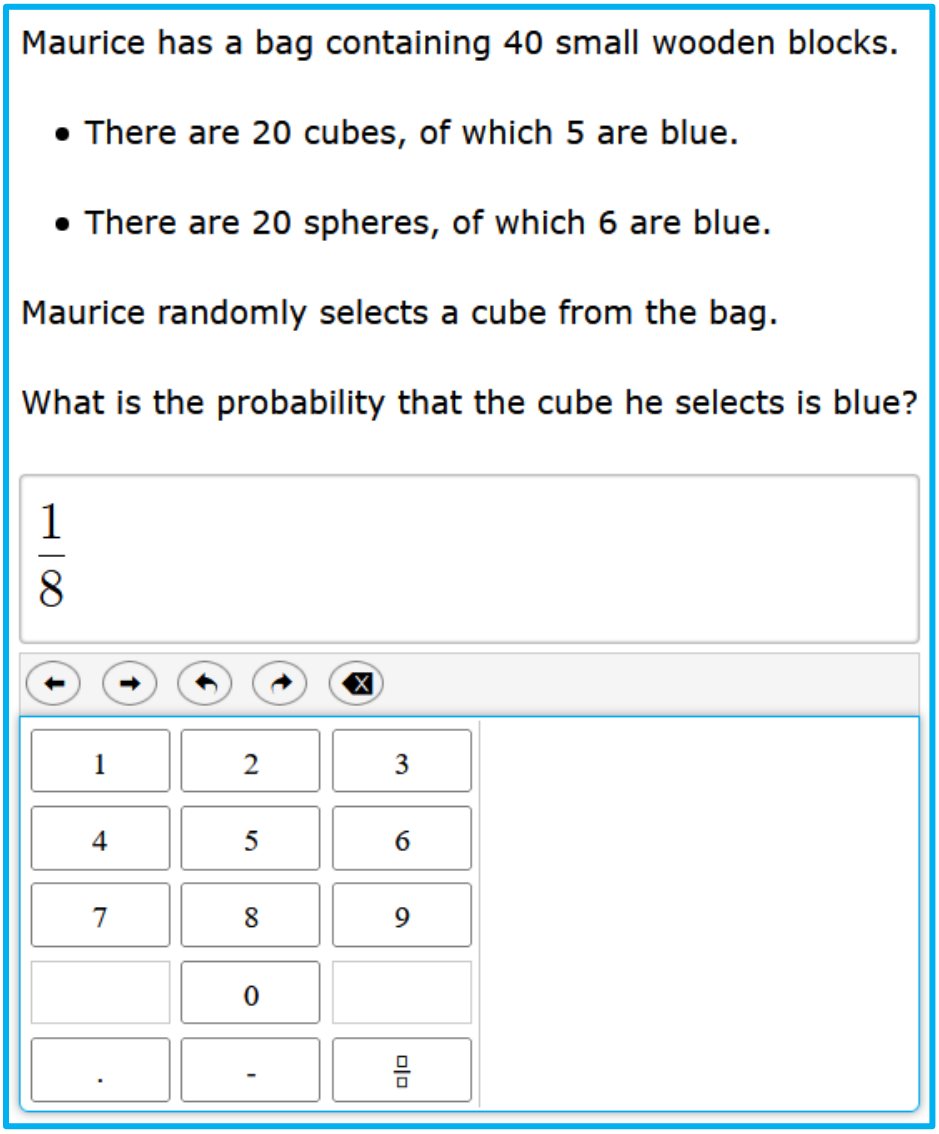



**Notes on Scoring**

This response earns no credit (0 points) because it shows an incorrect conditional probability.

The student may determine the probability of Maurice selecting a blue block (11) out of all blocks (40) in the bag, or $\frac{11}{40}$.

## Sample Response: 0 points




**Notes on Scoring**

This response earns no credit (0 points) because it shows an incorrect conditional probability.

The student may see that 5 of the cubes are blue and concludes that the possibility of drawing one of them from the entire bag ($20 + 20 = 40$ shapes) is $\frac{5}{40}$ or $\frac{1}{8}$.

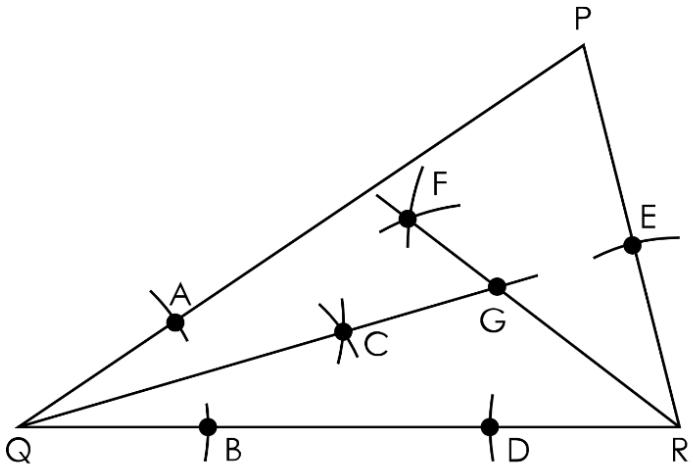
# Geometry
# Spring 2021 Item Release

# Question 42

# Question and Scoring Guidelines

## Question 42

The first few steps to construct a circle inscribed in triangle PRQ are shown.




Adam and his friend Jason make the following claims about how they can identify the next step in the construction.

- Adam: Use the fact that the radius drawn to the point of tangency is perpendicular to the tangent line of a circle.
- Jason: Use the fact that the radius drawn perpendicular to a chord bisects the chord.

Complete the sentence to create a statement about whose claim is correct.

[ ] is correct because this fact can be used to determine the length of the [ ] of the circle by constructing a perpendicular line from point [ ] to any of the sides of the triangle.

**Drop Down Choices:**

[ ] is correct because this fact can be used to determine the length

- Adam
- Jason

of the [ ] of the circle by constructing a perpendicular line

- chord
- diameter
- radius

from point [ ] to any of the sides of the triangle.

- C
- E
- G

**Points Possible:** 1

**Content Cluster:** Understand and apply theorems about circles.

**Content Standard:** Construct the inscribed and circumscribed circles of a triangle; prove and apply the property that opposite angles are supplementary for a quadrilateral inscribed in a circle. *(G.C.3)*

**Depth of Knowledge:** Level 3
a. Interpret information from a complex graph
f. Perform a procedure with multiple steps and multiple decision points
o. Analyze the similarities and differences between procedures

# Scoring Guidelines

Exemplar Response

- **Adam** is correct because this fact can be used to determine the length of the **radius** of the circle by constructing a perpendicular line from point **G** to any of the sides of the triangle.

Other Correct Responses

- N/A

For this item, a full-credit response includes:

- the correct statement (1 point).

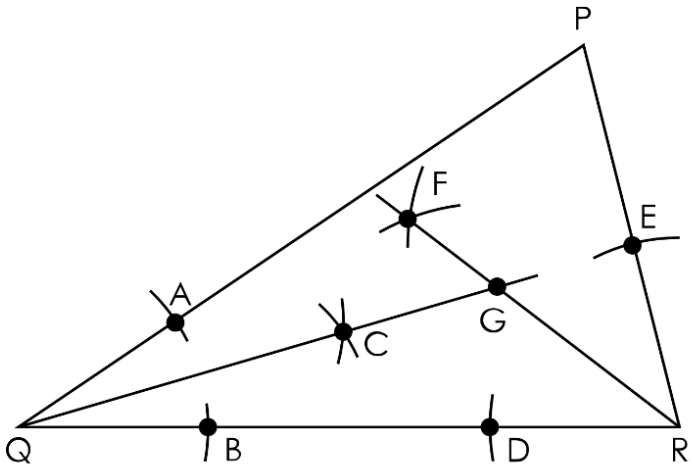
# Geometry
# Spring 2021 Item Release

# Question 42

# Sample Responses

## Sample Response: 1 point

The first few steps to construct a circle inscribed in triangle PRQ are shown.




Adam and his friend Jason make the following claims about how they can identify the next step in the construction.

- Adam: Use the fact that the radius drawn to the point of tangency is perpendicular to the tangent line of a circle.
- Jason: Use the fact that the radius drawn perpendicular to a chord bisects the chord.

Adam is correct because this fact can be used to determine the length of the radius of the circle by constructing a perpendicular line from point G to any of the sides of the triangle.

**Notes on Scoring**

This response earns full credit (1 point) because it correctly determines the next step in the construction and the justification for this step.

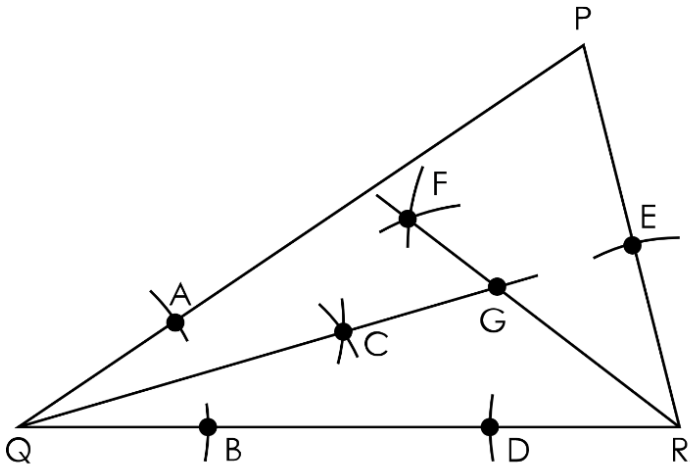
The student uses the fact that the center of the inscribed circle is a point, G, where three triangles' angle bisectors intersect. This point is equidistant from three sides.

A distance from the center of the inscribed circle to any side of the triangle is measured by the length of the perpendicular line segment going from the center onto the side of the triangle. This distance represents the radius of the inscribed circle. When the inscribed circle is constructed, the triangle and the circle share three points that are called points of tangency.

So, the next step in the construction is to construct a perpendicular line segment (a radius) from a point G to any side of the triangle at the point of tangency.

## Sample Response: 0 points

The first few steps to construct a circle inscribed in triangle PRQ are shown.




Adam and his friend Jason make the following claims about how they can identify the next step in the construction.

- Adam: Use the fact that the radius drawn to the point of tangency is perpendicular to the tangent line of a circle.
- Jason: Use the fact that the radius drawn perpendicular to a chord bisects the chord.

Jason is correct because this fact can be used to determine the length of the radius of the circle by constructing a perpendicular line from point G to any of the sides of the triangle.

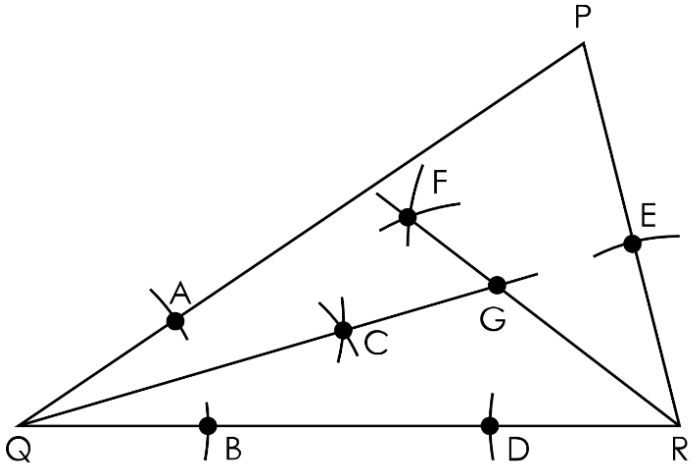
**Notes on Scoring**

This response earns no credit (0 points) because it incorrectly determines the next step in the construction and the justification for this step.

The student may confuse the characteristics of an inscribed circle and a circumscribed circle, not realizing that the point where the perpendicular bisectors intersect is the center of the circumscribed circle.

## Sample Response: 0 points

The first few steps to construct a circle inscribed in triangle PRQ are shown.




Adam and his friend Jason make the following claims about how they can identify the next step in the construction.

- Adam: Use the fact that the radius drawn to the point of tangency is perpendicular to the tangent line of a circle.
- Jason: Use the fact that the radius drawn perpendicular to a chord bisects the chord.

Jason is correct because this fact can be used to determine the length of the chord of the circle by constructing a perpendicular line from point G to any of the sides of the triangle.

**Notes on Scoring**

This response earns no credit (0 points) because it incorrectly determines the next step in the construction and the justification for this step.

The student may understand the need to construct a perpendicular line segment from a point G to the sides of the triangle but incorrectly thinks that this line segment will bisect the side. The student may also confuse the radius with the chord. Finding the radius of a circle should be the next step in the construction.

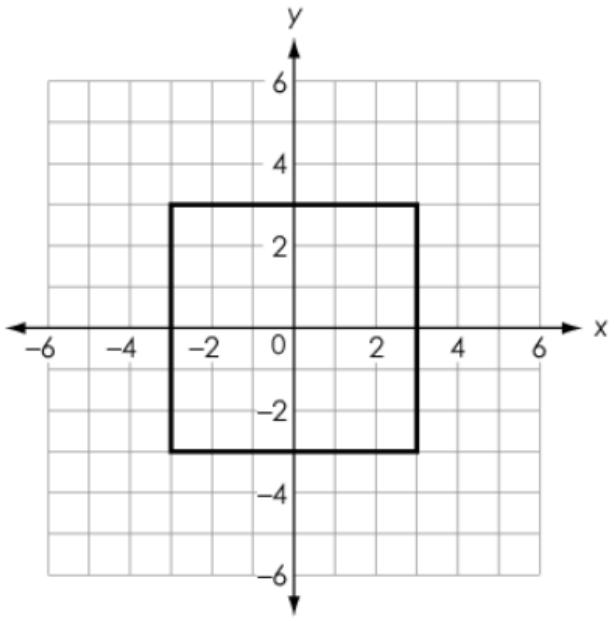
# Geometry
# Spring 2021 Item Release

# Question 43

# Question and Scoring Guidelines

# Question 43

A square is shown.




Select all of the transformations that carry the square onto itself.

- [ ] a reflection over the line $x = 3$
- [ ] a reflection over the line $y = -x$
- [ ] a 90-degree clockwise rotation about the origin
- [ ] a 360-degree clockwise rotation about the point $(-3, 3)$
- [ ] a 180-degree clockwise rotation about the point $(0, -3)$

**Points Possible:** 1

**Content Cluster:** Experiment with transformations in the plane.

**Content Standard:** Identify the symmetries of a figure, which are the rotations and reflections that carry it onto itself. *(G.CO.3)*

**Depth of Knowledge:** Level 2
b. Interpret information from a simple graph
e. Compare and/or contrast figures or statements
l. Select a procedure according to criteria and perform it

# Scoring Guidelines

Rationale for First Option: This is incorrect. The student may think that since the right side of the square is carried onto itself, the entire square is carried onto itself.

Rationale for Second Option: **Key** – The student correctly identifies that a reflection over the line $y = -x$ would carry the square onto itself because two vertices, (–3, 3) and (3, –3), remain unchanged and the vertex (3, 3) carries onto (–3, –3).

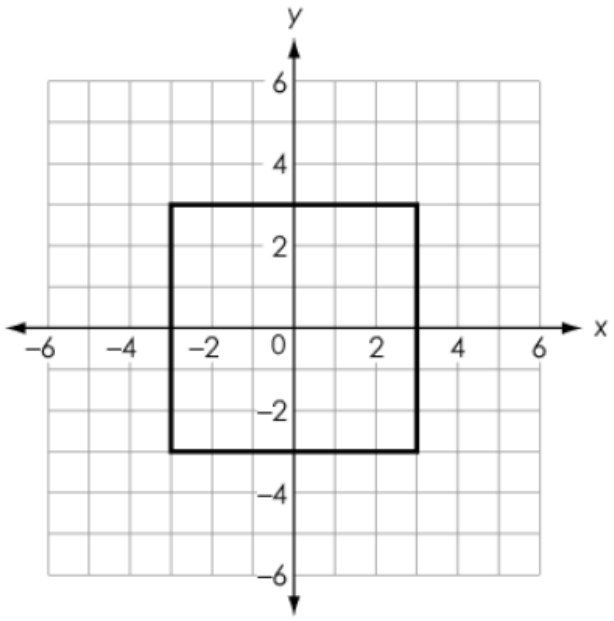
Rationale for Third Option: **Key** – The student correctly observes that since the square is centered at the origin, a 90-degree rotation about the origin would return the square to the original position on the coordinate grid and all the vertices would coincide.

Rationale for Fourth Option: **Key** – The student correctly observes that a 360-degree rotation about the point (–3, 3) would return the square to the original position on the coordinate grid and all the vertices would coincide.

Rationale for Fifth Option: This is incorrect. The student may think that since the bottom side of the square is carried onto itself, the entire square is carried onto itself.

## Sample Response: 1 point

A square is shown.




Select all of the transformations that carry the square onto itself.

- [ ] a reflection over the line $x = 3$
- [x] a reflection over the line $y = -x$
- [x] a 90-degree clockwise rotation about the origin
- [x] a 360-degree clockwise rotation about the point $(-3, 3)$
- [ ] a 180-degree clockwise rotation about the point $(0, -3)$

# Geometry
# Spring 2021 Item Release

# Question 45

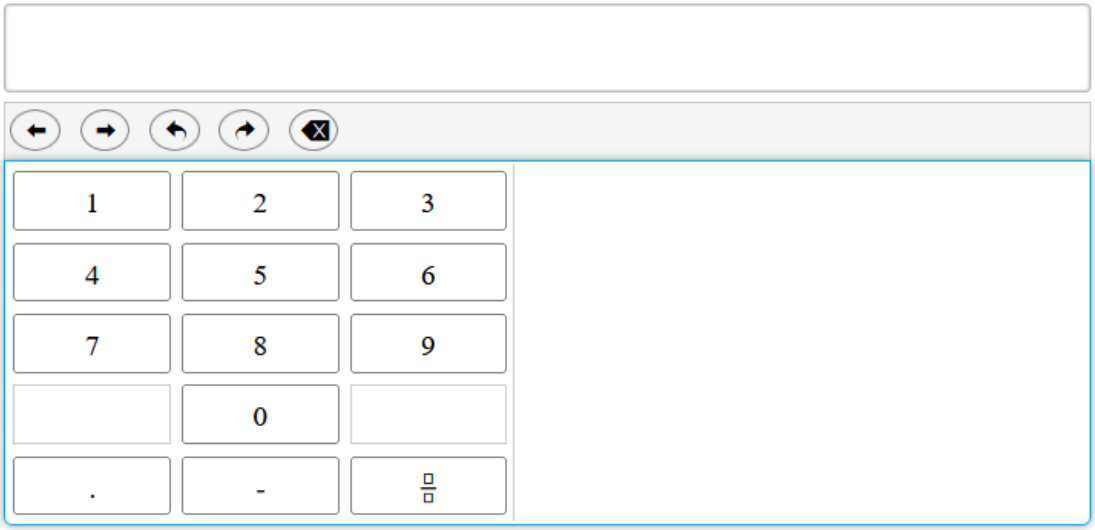
# Question and Scoring Guidelines

# Question 45

Isosceles triangle JKL has a perimeter of 32 units and the given vertices.

- J (−3, −9)
- K (−3, 6)
- L ($x$, −1.5)

What is a possible $x$-coordinate for point L?




**Points Possible:** 1

**Content Cluster:** Use coordinates to prove simple geometric theorems algebraically and to verify specific geometric statements.

**Content Standard:** Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.★ *(G.GPE.7)*

**Depth of Knowledge:** Level 3
b. Explain thinking when more than one response is possible
f. Perform a procedure with multiple steps and multiple decision points
i. Formulate a mathematical model for a complex situation

# Scoring Guidelines

Exemplar Response

- $-7$

Other Correct Responses

- 1, or any equivalent values

For this item, a full-credit response includes:

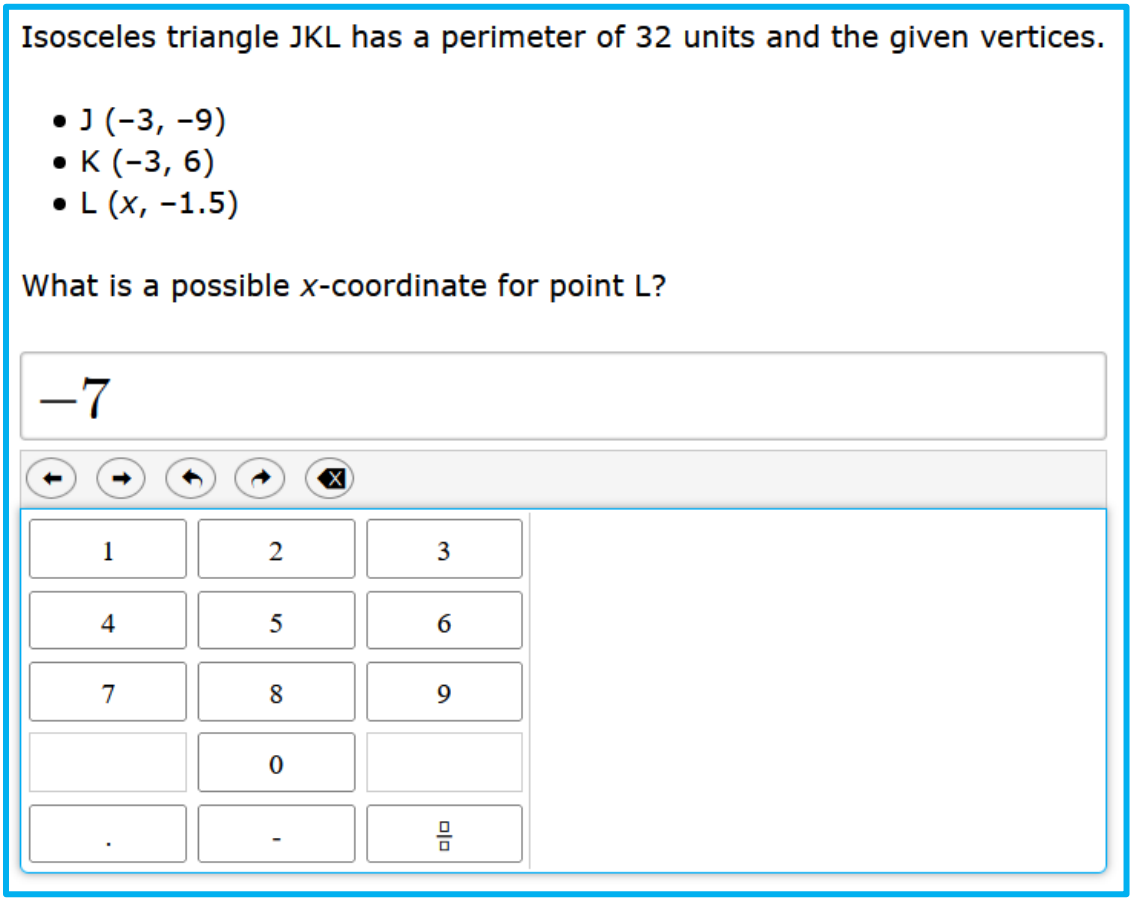
- a correct value (1 point).

# Geometry
# Spring 2021 Item Release

# Question 45

# Sample Responses

## Sample Response: 1 point



**Notes on Scoring**

This response earns full credit (1 point) because it shows a correct possible x-coordinate for the third vertex of the isosceles triangle.

To find the unknown coordinate for point L, first determine the length of one side, JL or KL, of the triangle JKL. Begin by using the definition of a perimeter.

The perimeter of a triangle JKL is the sum of three side lengths. Each side length can be found by substituting the coordinates of the end points into the Distance formula, $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$, two points at a time. The student calculates the length of side JK as $\sqrt{(-3 - (-3))^2 + (-9 - 6)^2} = 15$.

The perimeter of triangle JKL is 32, so the other two sides share 17 units, because $32 - 15 = 17$. Since the triangle is isosceles, these two side lengths are equal, and each side is 8.5 units.

Using the Distance formula again, the student may create the equation $8.5 = \sqrt{(-3 - x)^2 + (6 - (-1.5))^2}$ and then solve it for x:

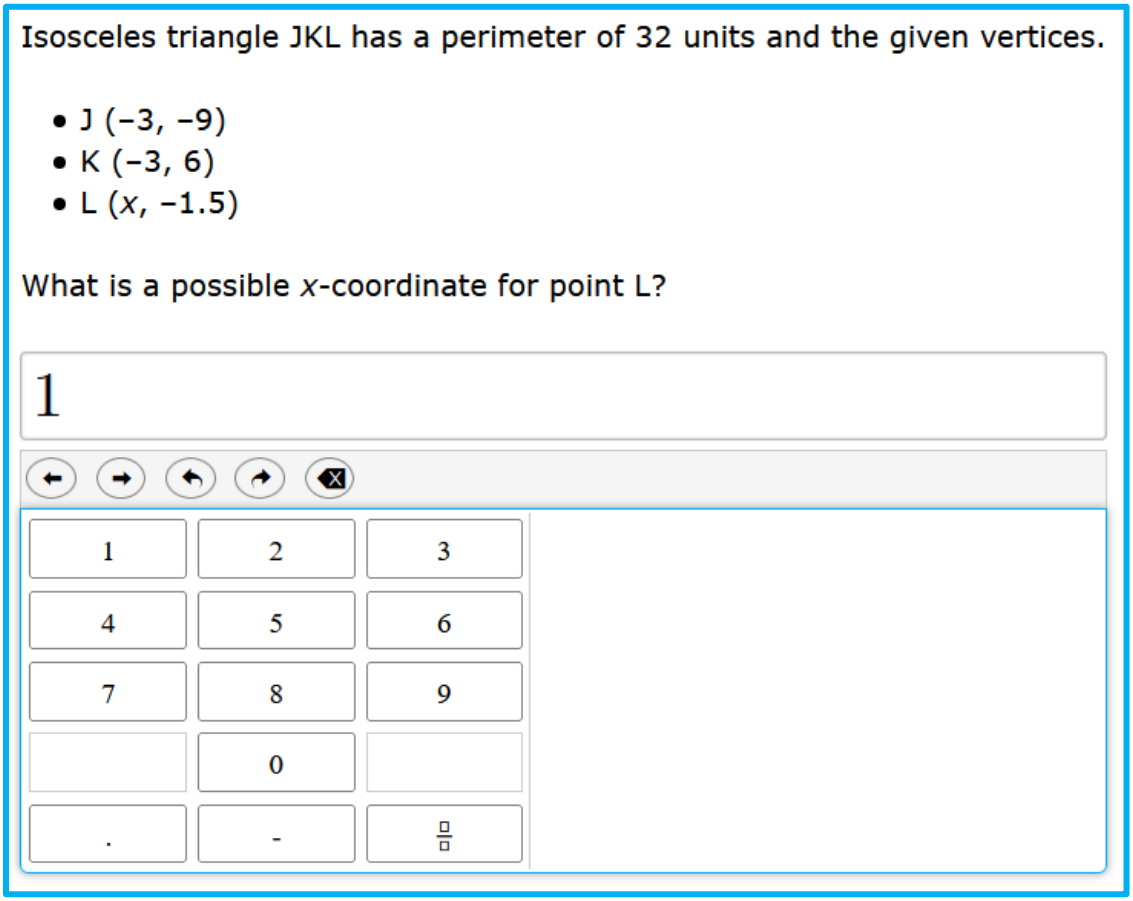
$$
\begin{aligned}
8.5 &= \sqrt{(-3 - x)^2 + (6 - (-1.5))^2} \\
8.5 &= \sqrt{(-3 - x)^2 + (6 - (-1.5))^2} \\
72.25 &= (-3 - x)^2 + (6 - (-1.5))^2 \\
72.25 &= (-3 - x)^2 + 7.5^2 \\
72.25 &= (-3 - x)^2 + 56.25 \\
16 &= (-3 - x)^2 \\
\sqrt{16} &= \sqrt{(-3 - x)^2} \\
\pm 4 &= -3 - x
\end{aligned}
$$

$$
\begin{aligned}
4 &= -3 - x \quad \text{or} \quad -4 = -3 - x \\
4 + 3 &= -x \qquad\quad -4 + 3 = -x \\
7 &= -x \qquad\quad\;\; -1 = -x \\
x &= -7 \qquad\quad\;\; x = 1
\end{aligned}
$$

Either value is an acceptable possible x-coordinate for point L.

The student chooses –7 as the x-coordinate for point L.

## Sample Response: 1 point



**Notes on Scoring**

This response earns full credit (1 point) because it shows a correct possible x-coordinate for the third vertex of the isosceles triangle.

To find the unknown coordinate for point L, first determine the length of one side, JL or KL, of the triangle JKL. Begin by using the definition of a perimeter.

The perimeter of a triangle JKL is the sum of three side lengths. Each side length can be found by substituting the coordinates of the end points into the Distance formula, $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = 15$, two points at a time. The student calculates the length of side JK as $\sqrt{(-3 - (-3))^2 + (-9 - 6)^2} = 15$. The perimeter of triangle JKL is 32, so the other two sides share 17 units, because $32 - 15 = 17$. Since the triangle is isosceles, these two side lengths are equal, and each side is 8.5 units.

Using the Distance formula again, create the equation $8.5 = \sqrt{(-3 - x)^2 + (6 - (-1.5))^2}$ and then solve it for x:

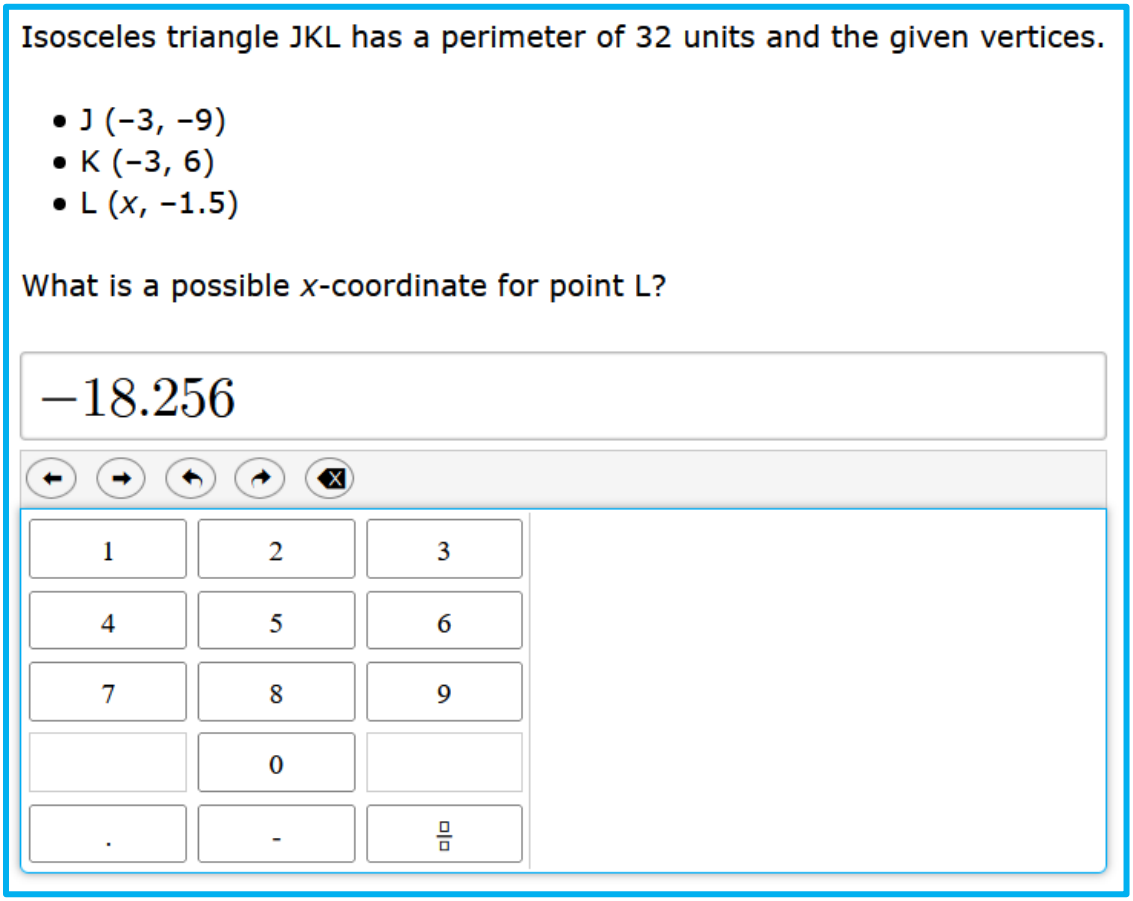
$$
\begin{aligned}
8.5 &= \sqrt{(-3-x)^2 + (6-(-1.5))^2} \\
72.25 &= (-3-x)^2 + (6-(-1.5))^2 \\
72.25 &= (-3-x)^2 + 7.5^2 \\
72.25 &= (-3-x)^2 + 56.25 \\
16 &= (-3-x)^2 \\
\sqrt{16} &= \sqrt{(-3-x)^2} \\
\pm 4 &= -3 - x
\end{aligned}
$$

$$
\begin{aligned}
4 &= -3 - x \quad \text{or} \quad -4 = -3 - x \\
4 + 3 &= -x \qquad -4 + 3 = -x \\
7 &= -x \qquad -1 = -x \\
x &= -7 \qquad x = 1
\end{aligned}
$$

Either value is an acceptable possible x-coordinate for point L.

The student chooses 1 as the x-coordinate for point L.

## Sample Response: 0 points



**Notes on Scoring**

This response earns no credit (0 points) because it shows an incorrect possible x-coordinate for the third vertex of the isosceles triangle.

To find the unknown coordinate for point L, the student may first determine the length of one side, JL or KL, of the triangle JKL using the definition of perimeter first.

The perimeter of a triangle JKL is the sum of three side lengths. Each side length can be found by substituting the coordinates of the end points into the Distance formula, $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$, two at a time. The student calculates the length of side JK as $\sqrt{(-3 - (-3))^2 + (-9 - 6)^2} = 15$.

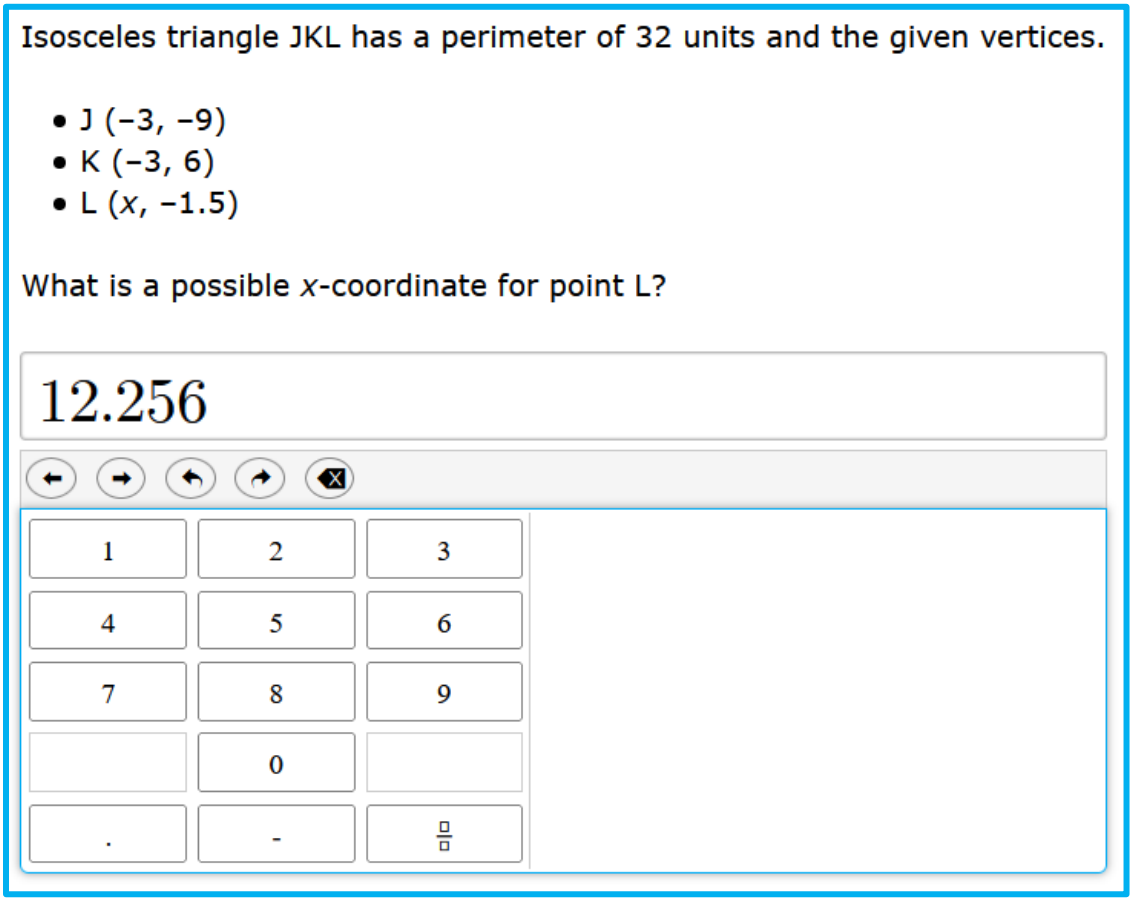
The perimeter of triangle JKL is 32, so the other two sides share 17 units, because $32 - 15 = 17$. The student may neglect that the triangle is isosceles and uses 17 to find the missing coordinate for point L using the Distance formula $17 = \sqrt{(-3 - x)^2 + (6 - (-1.5))^2}$ and then solve it for x:

$$
\begin{aligned}
17 &= \sqrt{(-3-x)^2 + (6-(-1.5))^2} \\
289 &= (-3-x)^2 + (6 - (-1.5))^2 \\
289 &= (-3-x)^2 + 7.5^2 \\
289 &= (-3-x)^2 + 56.25 \\
232.75 &= (-3-x)^2 \\
\sqrt{232.75} &= \sqrt{(-3-x)^2} \\
\pm 15.256 &\approx -3-x
\end{aligned}
$$

$$
\begin{aligned}
15.256 &\approx -3 - x \quad \text{or} \quad -15.256 \approx -3 - x \\
15.256 + 3 &\approx -x \qquad\qquad -15.256 + 3 \approx -x \\
18.256 &\approx -x \qquad\qquad -12.256 \approx -x \\
x &\approx -18.256 \qquad\qquad x \approx 12.256
\end{aligned}
$$

The student chooses –18.256 as the x-coordinate for point L.

## Sample Response: 0 points



**Notes on Scoring**

This response earns no credit (0 points) because it shows an incorrect possible x-coordinate for the third vertex of the isosceles triangle.

To find the unknown coordinate for point L, the student may first determine the length of one side, JL or KL, of the triangle JKL using the definition of perimeter first.

The perimeter of a triangle JKL is the sum of three side lengths. Each side length can be found by substituting the coordinates of the end points into the Distance formula, $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$, two at a time. The student calculates the length of side JK as $\sqrt{(-3 - (-3))^2 + (-9 - 6)^2} = 15$.

The perimeter of triangle JKL is 32, so the other two sides share 17 units, because $32 - 15 = 17$. The student may neglect that the triangle is isosceles and uses 17 to find the missing coordinate for point L using the Distance formula $17 = \sqrt{(-3 - x)^2 + (6 - (-1.5))^2}$ and then solve it for x:

$$
\begin{aligned}
17 &= \sqrt{(-3-x)^2 + (6-(-1.5))^2} \\
289 &= (-3-x)^2 + (6 - (-1.5))^2 \\
289 &= (-3-x)^2 + 7.5^2 \\
289 &= (-3-x)^2 + 56.25 \\
232.75 &= (-3-x)^2 \\
\sqrt{232.75} &= \sqrt{(-3-x)^2} \\
\pm 15.256 &\approx -3-x
\end{aligned}
$$

$$
\begin{aligned}
15.256 &\approx -3 - x \quad \text{or} \quad -15.256 \approx -3 - x \\
15.256 + 3 &\approx -x \qquad\qquad -15.256 + 3 \approx -x \\
18.256 &\approx -x \qquad\qquad -12.256 \approx -x \\
x &\approx -18.256 \qquad\qquad x \approx 12.256
\end{aligned}
$$

The student chooses 12.256 as the x-coordinate for point L.

# Geometry
# Spring 2021 Item Release

# Question 47

# Question and Scoring Guidelines

# Question 47

Triangle ABC lies on the coordinate plane with vertices located at A $(8, 6)$, B $(2, -5)$, and C $(-5, 1)$. The triangle is transformed using the rule $(x, y) \rightarrow (x + 3, 2y)$ to create triangle A'B'C'.

Determine the coordinates of triangle A'B'C'.

| | | | |
|---|---|---|---|
| A' ( | | , | ) |
| B' ( | | , | ) |
| C' ( | | , | ) |

**Points Possible:** 1

**Content Cluster:** Experiment with transformations in the plane.

**Content Standard:** Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch). *(G.CO.2)*

**Depth of Knowledge:** Level 2
j. Translate between tables, graphs, words and symbolic notation
l. Select a procedure according to criteria and perform it

# Scoring Guidelines

Exemplar Response

| | | | | |
|---|---|---|---|---|
| A' ( | 11 | , | 12 | ) |
| B' ( | 5 | , | –10 | ) |
| C' ( | –2 | , | 2 | ) |

Other Correct Responses

- N/A

For this item, a full-credit response includes:

- the correct coordinates (1 point).

# Geometry
# Spring 2021 Item Release

# Question 47

# Sample Responses

## Sample Response: 1 point

Triangle ABC lies on the coordinate plane with vertices located at A $(8, 6)$, B $(2, -5)$, and C $(-5, 1)$. The triangle is transformed using the rule $(x, y) \rightarrow (x + 3, 2y)$ to create triangle A'B'C'.

Determine the coordinates of triangle A'B'C'.

| | | | | |
|---|---|---|---|---|
| A' ( | 11 | , | 12 | ) |
| B' ( | 5 | , | -10 | ) |
| C' ( | -2 | , | 2 | ) |

**Notes on Scoring**

This response earns full credit (1 point) because it shows three correct coordinate pairs for the transformed triangle.

The rule $(x, y) \rightarrow (x + 3, 2y)$ describes a transformation of the original triangle ABC to create triangle A'B'C'. It shows that 3 is added to the *x*-coordinate and 2 is multiplied by the *y*-coordinate of each given vertex.

Following this rule, the student performs the following calculations:

$A\ (8, 6) \rightarrow A'\ (8 + 3, 2 \bullet 6)$ or A'(11, 12)
$B\ (2, -5) \rightarrow B'\ (2 + 3, 2 \bullet (-5))$ or B'(5, −10)
$C\ (-5, 1) \rightarrow C'\ (-5 + 3, 2 \bullet 1)$ or C'(−2, 2)

## Sample Response: 0 points

Triangle ABC lies on the coordinate plane with vertices located at A $(8, 6)$, B $(2, -5)$, and C $(-5, 1)$. The triangle is transformed using the rule $(x, y) \rightarrow (x + 3, 2y)$ to create triangle A'B'C'.

Determine the coordinates of triangle A'B'C'.

| | | | | |
|---|---|---|---|---|
| A' ( | 11 | , | 6 | ) |
| B' ( | 5 | , | -5 | ) |
| C' ( | -2 | , | 1 | ) |

**Notes on Scoring**

This response earns no credit (0 points) because it shows three incorrect coordinate pairs for the transformed triangle.

The student may apply only the x-coordinate portion of the rule and neglect to apply the *y*-coordinate portion of the rule.

## Sample Response: 0 points

Triangle ABC lies on the coordinate plane with vertices located at A $(8, 6)$, B $(2, -5)$, and C $(-5, 1)$. The triangle is transformed using the rule $(x, y) \rightarrow (x + 3, 2y)$ to create triangle A'B'C'.

Determine the coordinates of triangle A'B'C'.

| | | | | |
|---|---|---|---|---|
| A' ( | 8 | , | 12 | ) |
| B' ( | 2 | , | -10 | ) |
| C' ( | -5 | , | 2 | ) |

**Notes on Scoring**

This response earns no credit (0 points) because it shows three incorrect coordinate pairs for the transformed triangle.

The student may apply only the *y*-coordinate portion of the rule and neglect to apply the x-coordinate portion of the rule.

# Geometry
# Spring 2021 Item Release

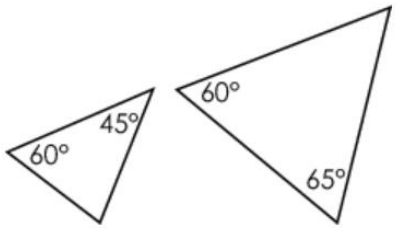
# Question 48

# Question and Scoring Guidelines

# Question 48

Which pair of triangles is similar?

Ⓐ

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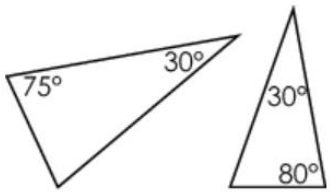


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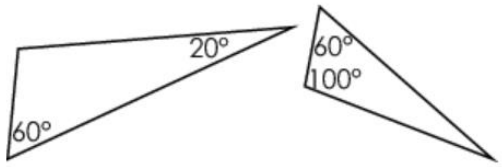


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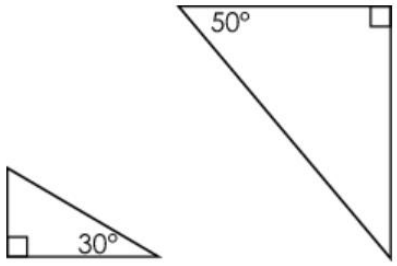



**Points Possible:** 1

**Content Cluster:** Understand similarity in terms of similarity transformations.

**Content Standard:** Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar. *(G.SRT.3)*

**Depth of Knowledge:** Level 1
a. Recall, observe, or recognize a fact, definition, term, or property
j. Retrieve information from a table or graph

# Scoring Guidelines

Rationale for Option A: This is incorrect. The student may notice that both triangles have a 60-degree angle and think the triangles are similar. The student may not realize that for triangles to be similar, a second pair of angles must have equal measures.

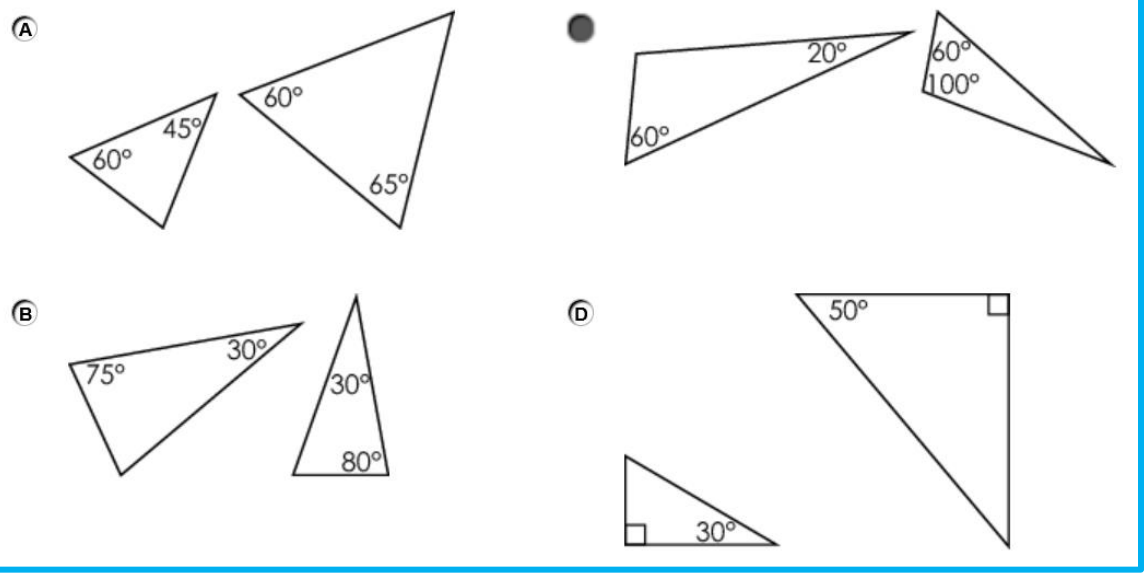
Rationale for Option B: This is incorrect. The student may see that both triangles have a 30-degree angle and think the triangles are similar. The student may not realize that for triangles to be similar, a second pair of angles must have equal measures.

Rationale for Option C: **Key** – The student identifies the correct pair of similar triangles based on the AA criterion. The missing angle in the left triangle has a measure of 100 degrees (180 – (60 + 20)) and the missing angle in the right triangle has a measure of 20 degrees (180 – (60 + 100)). There are three pairs of corresponding angles that have equal measures; therefore, the triangles are similar.

Rationale for Option D: This is incorrect. The student may think all right triangles are similar and do not confirm that there should be a second pair of angles with equal measures.

## Sample Response: 1 point

Which pair of triangles is similar?



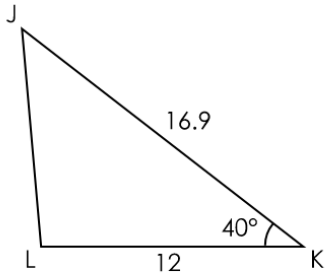
# Geometry
# Spring 2021 Item Release

# Question 49

# Question and Scoring Guidelines

# Question 49

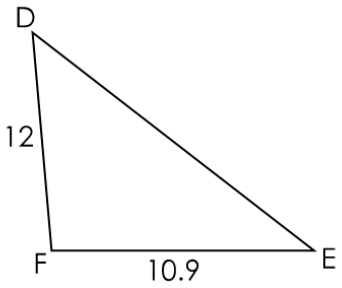
Triangle JKL is shown.




Triangle DEF is congruent to triangle JKL.

Which triangle could be DEF?

Ⓐ

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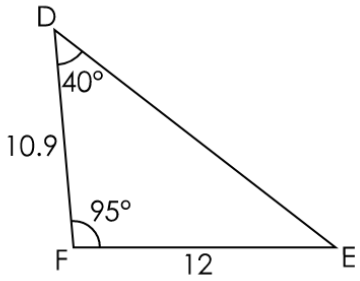


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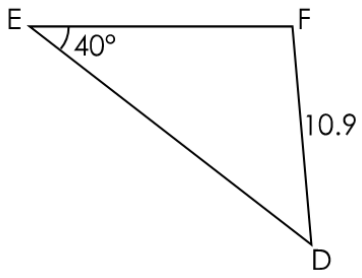


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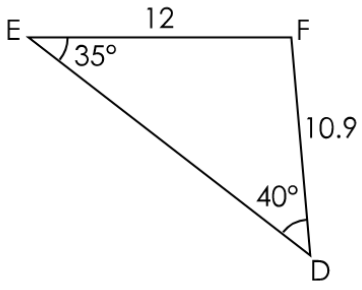



**Points Possible:** 1

**Content Cluster:** Understand congruence in terms of rigid motions.

**Content Standard:** Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions. *(G.CO.8)*

**Depth of Knowledge:** Level 1
a. Recall, observe, or recognize a fact, definition, term, or property
j. Retrieve information from a table or graph

# Scoring Guidelines

Rationale for Option A: This is incorrect. The student may think that since the triangles have one pair of equal side lengths, the triangles must be congruent but miss that, given the mapping, side KL should be congruent to side EF, which is not the case for these triangles because those sides have different lengths.

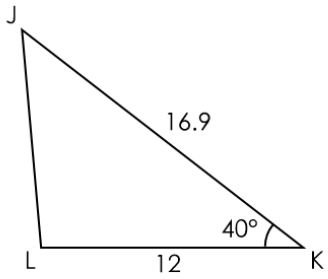
Rationale for Option B: This is incorrect. The student may not realize that angle E must be congruent to angle K given the mapping and miss that given the other angle measures in DEF, angle E is 45 degrees, not 40 degrees.

Rationale for Option C: **Key** – The student selects a possible triangle DEF, as there is nothing in the given information that precludes the specified mapping because angle E is congruent to angle K and JK is not specified.

Rationale for Option D: This is incorrect. The student may not realize that angle E must be congruent to angle K given the mapping and think that as long as there is a pair of congruent sides and a pair of 40-degree angles, the triangles are congruent.

## Sample Response: 1 point

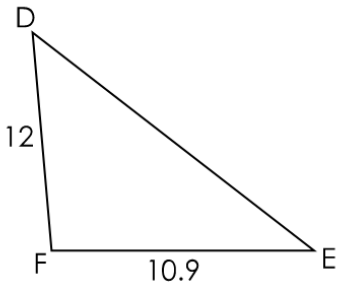
Triangle JKL is shown.




Triangle DEF is congruent to triangle JKL.

Which triangle could be DEF?

Ⓐ

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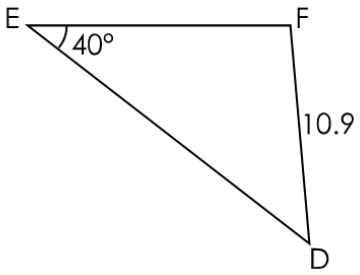


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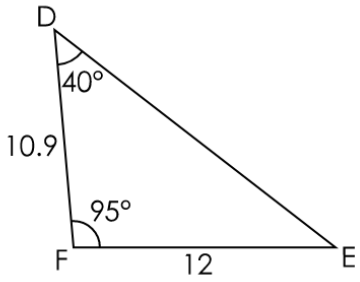


Ⓓ